NUMERICAL DETERMINATION OF J-R CURVE USING VOID MODEL

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The paper presents the determination of the J-R curve for an aluminium alloy. J-integral value is determined by numerical simulation of the test prescribed in ASTM E 1820-01. The crack growth Δa corresponding to the load increment is determined numerically. The Complete Gurson Model (CGM) based upon void models is adopted as a constitutive relation. This model describes the nucleation, growth and the coalescence of voids and it considers the constraint effect as well. The advantage of this approach is when compared with the standard experimental determination of the J-R curve that the number of experiments is reasonably lower in this case since only the experiments for the determination of the CGM parameters are needed. These experiments require much less effort and are less time demanding. Numerical simulations for the determination of the J-R curve were performed for the CT specimen. The finite element method (FEM) code ABAQUS was used for the computation. A subroutine including the CGM was implemented into the code. Static loading and room temperature were assumed in all simulations.

Key words: aluminium, J-R curve, numerical simulation, Complete Gurson Model, CT specimen

1 INTRODUCTION

The determination of the J-R curves based upon the ASTM E 1820-01 standard is rather time consuming and requires number of experiments. A minimum of 15 specimens must be tested in order to obtain a reliable J-R curve. Effort has been made to obtain the J-R curves by a less complicated experiment combined with numerical simulations. One of these methods is the Single-sample J-integral Test. For this fracture testing the on-line or continuous crack monitoring is required. This is generally performed by the Unloading Compliance Measurement (ASTM E 813-87) or by Electric Potential Drop methods. Another possibility for the determination of the J-R curve is the Load Normalization Technique. This technique does not require the on-line monitoring and it is based on the principle of load separation. The load may be mathematically expressed as a function of the crack length and the plastic deformation. The evaluation procedure uses the load-line displacement diagrams (see e.g. 3). Another method used for the determination of the J-R curve is the Direct Method 1, which employs the load-load point displacement records of a single specimen test in the procedure. Then, the Direct Method Program, including necessary input data, calibration functions and elastic-plastic analysis, is applied. The advantages of this procedure are that it avoids the accuracy problems when measuring the crack growth, and the reduction of the testing time. Some results are presented e.g. in 1. The possibility to determine the J-R curve without testing the standard specimens is discussed in this paper. Such approach requires only simple tensile tests and proper numerical simulations.

2 EXPERIMENTAL DETERMINATION OF J-R CURVE

The J-R curve can be determined experimentally according to the ASTM standard. This procedure requires large number of experiments fulfilling the assumptions for the specimens used and for the measurement procedures. Three types of specimens are used: single edge bend specimen, compact specimen and disc-shaped compact specimen with fatigue pre-crack. The J-R curve determination procedure consists of loading the specimen to a given level and then determining the J-integral value and the crack length...
increment $\Delta a$. Thus, one point of the $J$-$R$ curve is obtained. The minimum number of points to be determined is 15, whereas each point must be obtained from measurement on a new specimen subjected to higher load level, thus exhibiting longer crack length. In order to measure the crack length, each specimen must be broken using brittle fracture after cooling.

Since the numerical determination of the $J$-$R$ curve in this work is performed by the simulation of experiment, the basic relations for the $J$-integral determination and the procedure of the relevant crack length increment determination necessary for the experiment evaluation are presented below. CT specimen is used.

For the $J$-integral determination so-called load-line displacement curve ($P$ versus $v$) is used. A typical curve is shown in Figure 1a, where $A_{pl}$ represents the area corresponding to plastic work. The $J$-integral is then determined as:

$$ J = \frac{K^2 (1 - \nu^2)}{E} + J_{pl} $$

where the first part of equation (1) represents the elastic value of $J$-integral, $K$ is the stress intensity factor, $\nu$ is Poisson’s ratio, $E$ is Young’s modulus, and $J_{pl}$ represents the $J$-integral plastic component. The stress intensity factor is determined for the relevant force $P_i$ as:

$$ K_i = \frac{P_i}{B B_N W^{1/2}} \int \frac{a_i}{W} $$

where $f(a/W)$ is the compliance function, $a_i$ is the crack length, $W$ is the specimen width, $B$ is the specimen thickness and $B_N$ is the specimen thickness measured at side grooves. The $J$-integral plastic component is determined from the loading-force plastic work as:

$$ J_{pl(i)} = J_{pl(i-1)} + \left( \frac{\eta_{i-1}}{b_{i-1}} \right) \frac{A_{pl(i)} - A_{pl(i-1)}}{B_N} - \gamma_{i-1} \frac{a_i - a_{i-1}}{b_{i-1}} $$

where

$$ \eta_{i-1} = 2.0 + 0.522 b_{i-1} / W $$

$$ \gamma_{i-1} = 1.0 + 0.76 b_{i-1} / W $$

The crack length increment is measured optically. After loading to the given level the specimen is unloaded and the free crack surfaces are heat tinted. Then the specimen is cooled and brittle broken. Figure 2 shows the surfaces of the CT specimen after brittle fracture and 9 locations for the crack increment measurement.

The crack length increment value is determined as:

$$ \Delta a = \frac{1}{8} \left( \Delta a_1 + \Delta a_2 + \sum_{i=2}^{8} \Delta a_i \right) $$

where $\Delta a_i = a_i - a_0$. The corresponding values of $J$ and $\Delta a$ are recorded into the diagram together with the construction lines, offset line and two exclusion lines, which define points from which the $J$-$R$ curve will be determined (see Figure 3). The $J$-$R$ curve is then the power law regression line:

$$ \ln J = \ln C_1 + C_2 \ln \left( \frac{\Delta a}{k} \right) $$

determined from points between two exclusion lines, where $k = 1$ mm (or $k = 0.0394$ in). Constants $C_1$ and $C_2$ are determined with the regression analysis. Value $J_0$ is given by the $J$-$R$ curve and the offset line crossing point, $\Delta a_{\text{min}}$ and $\Delta a_{\text{lim}}$ are given by the $J$-$R$ curve and the exclusion lines crossing points. The values $\Delta a_{\text{min}}$ and $\Delta a_{\text{lim}}$ and the exclusion lines then determine the feasible domain of the $J$ values.

Figure 1: a) Load – line displacement curve ($P$ versus $v$), b) Plastic work $\Delta A_{pl(i)}$ increment
Slika 1: a) Odvisnost obremenitve-premik obremenitvene črte ($P$ v odvisnosti od $v$); b) Poveček plastičnega dela $\Delta A_{pl(i)}$

Figure 2: Fracture surfaces (A – brittle breaking surface, B – crack increment, C – pre-cracked surface, D – notch surface) and 9 points for the crack length increment measurement
Slika 2: Površine preloma (A – površina krhkega preloma, B – poveček propagacije razpoke, C – začetna površina razpoke, D – površina zareze) in 9 točk meritev dolžine razpoke

Figure 3: Determination of the $J$-$R$ curve according to ASTM standard
Slika 3: Določitev krivulje $J$-$R$ po ASTM standardu
3 COMPUTATIONAL MODEL

Since the experimental determination of the J-R curve is not simple, other possibilities for the determination of this curve are sought. One of these possibilities is a numerical simulation of the test prescribed in ASTM standard. A proper model describing the real behaviour of the material during damage (the crack propagation) is necessary in order to obtain a reliable J-R curve. Since the stress-strain field near the crack tip cannot be described by common constitutive relations the material model used must include e.g. the effect of triaxiality, the nucleation, growth and coalescence of voids existing in the zone with high stress concentration. The above requirements are fulfilled by certain so-called void models, where the dilatation plasticity is used. This means that the plasticity condition is also function of the mean normal stress and that the volume change takes place, which is not considered in classical plasticity models.

One of the real plasticity models is the Complete Gurson Model (CGM), which eliminates the disadvantages of the Gurson Model (GM) and also of the modified Gurson Model (GTN) implemented by Tvergaard and Needleman.

The CGM yield function has the same form as that of GTN model:

\[
\phi(\sigma_e, \sigma_M, \sigma_k, f^*) = \frac{\sigma_e}{\sigma_M} + 2q_1 f^* \cos h \left( \frac{q_2 \sigma_k}{2\sigma_M} \right) \left[ 1 - (q_1 f^*) \right] = 0 \quad (7)
\]

where \( \sigma_e \) is the macroscopic von Mises equivalent stress, \( \sigma_M \) is the actual yield stress of the matrix material, \( \sigma_k \) is the trace of the macroscopic Cauchy stress tensor, \( f^* \) is the modified void volume fraction, \( q_1, q_2 \) are constants introduced by Tvergaard. This model involves phases of nucleation, growth and coalescence of voids. Contrary to the GTN model, for which 6 parameters must be determined experimentally, only 4 parameters (\( f_0 \) initial void volume fraction, \( \varepsilon_N \) mean nucleation equivalent plastic strain, \( s \) standard deviation and \( f_N \) volume fraction of void nucleating particles) are to be determined. It is particularly important that it is not necessary to determine experimentally the critical void volume fracture \( f_c \), which was considered to be a material constant in GTN and, as a matter of fact, it depends also on the stress state. Zhang eliminated this disadvantage by the introduction of the Thomason plastic limit load model into GTN and by this way the CGM was created. Parameters \( f_0, \varepsilon_N, s \) and \( f_N \) are determined by numerical fitting to the experimental data, where for the notched bar subjected to tension the relation load \( P \) versus notch diameter reduction \( \Delta d \) is recorded. Other CGM parameters are assumed to be \( q_1 = 1.5, q_2 = 1 \).

CGM enables the numerical simulation of the crack growth and the determination of the crack length increment \( \Delta a \) as well as the simulation of the load-line displacement curve \( P \) versus \( \nu \) (Figure 1), from which the relevant value of the J-integral can be determined using relations in equation (1) to equation (4).

4 J-R CURVE NUMERICAL SIMULATION

The problem was solved using the finite element method programme ABAQUS, into which the CGM was implemented as a subroutine. Static loading and room temperature were assumed in the analysis. The tested material was aluminium alloy with material constants: Young’s modulus \( E = 6.9122 \times 10^4 \) MPa, Poisson’s ratio \( v = 0.315 \), yield stress \( \sigma_y = 280 \) MPa. The true stress-strain diagram was available, too. For the CGM the following parameter values were used: \( q_1 = 1.5, q_2 = 1, f_0 = 0.02, \varepsilon_N = 0.1, s = 0.1, f_N = 0.01 \). As stated above, these parameters were obtained by the numerical simulation of a tensile test of a notched bar of circular cross-section. Numerical fitting to the experimental data of the loading force \( P \) versus diameter reduction \( \Delta d \) was carried out.
The numerical simulation of the crack growth was performed for CT specimen having dimensions as given in Figure 4. The computational model consisting of isoparametric 8-node elements of solid type is shown in Figure 5.

The specimen was loaded step-by-step. For each value of the loading force the relevant values of J-integral and the crack length increment were determined. The J-integral value can be numerically determined by several ways. In some cases the problem is solved as a plane strain problem and the J-integral is determined using the relation introduced by Rice (contour integral). Another possibility is using the domain integral introduced by Parks and modified by De Lorenzi. Taking into account and our own experience the J-integral value was obtained by the simulation of the experiment given by ASTM standard (see , part. 2). In the same way the crack length increment Δa was determined.

The J-R curve obtained by the above procedure is presented in Figure 6. According to ASTM standard the value of Jc was found to be 28155 J/m².

To obtain the J-R curve between exclusion lines the data from 40 points were used. From equation (6) and for k = 1 mm the relation

$$J = C_1 (\Delta a)^{C_2}$$  \hspace{1cm} (8)

where $C_1 = 36012.392$ and $C_2 = 0.178$, was obtained.

5 CONCLUSIONS

The presented work presents an approach for the numerical determination of the J-R curve. The method – numerical simulation of the experiment for the J-R curve determination according to ASTM E standard – was applied to the aluminium alloy.

As a constitutive relation the Complete Gurson Model was used. It is shown that using this model the real behaviour of ductile fracture can be described. The advantage of this method is that for the determination of the J-R curve only two types of simple experiments are needed. One for obtaining the stress-strain diagram, which characterises the material behaviour in both the elastic and plastic regions, and, the latter for obtaining the relation loading force P versus diameter reduction Δd for the bar of circular cross-section subjected to tensile load. From this relation the necessary parameters for CGM are obtained. It is evident that these experiments can be performed using simple equipment.

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6 REFERENCES

2. ASTM E 1820-01 Standard Test Method Measurement of Fracture Toughness

List of used symbols

- \( a_0 / m \) – original crack length
- \( a_i / m \) – current crack length
- \( A_{pl} / J \) – area corresponding to plastic work
- \( \Delta a / m \) – crack length increment
- \( \Delta a_{min}, \Delta a_{limit} / m \) – crack lengths determining feasible domain of the \( J \) values
- \( J / (J/m^2) \) – \( J \)-integral
- \( J_{pl} / (J/m^2) \) – \( J \)-integral plastic component
- \( K / (MPa \cdot m^{1/2}) \) – stress intensity factor
- \( B / m \) – specimen thickness
- \( W / m \) – specimen width
- \( B_N / m \) – specimen thickness
- \( \Delta d / m \) – notch diameter reduction
- \( f \) – void volume fraction
- \( q_1, q_2 \) – constants introduced by Tvergaard
- \( s \) – standard deviation
- \( E / Pa \) – Young’s modulus
- \( v / m \) – (load-line) displacement
- \( P / N \) – force
- \( v_{pl} / m \) – plastic part of load-line displacement
- \( \sigma_e / Pa \) – macroscopic von Mises equivalent stress
- \( \sigma_k^k / Pa \) – trace of the macroscopic Cauchy stress tensor
- \( \sigma_M / Pa \) – actual yield stress of the matrix material
- \( \sigma_y / Pa \) – yield stress
- \( v \) – Poisson’s ratio