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A NEW VERSION OF THE THEORY OF DUCTILITY AND CREEP UNDER CYCLIC LOADING

NOVA VERZIJA TEORIJE O DUKTILNOSTI IN LEZENJU PRI CIKLIČNI OBREMENITVI

Leonid B. Getsov[1], M. G. Kabelevskiy[2]

[1]St. Petersburg State Technical University, Russia
[2]ZNIITMASh, Moscow, Russia
guetsov@online.ru

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The model of deformation for isotropic metallic materials aimed at obtaining an increased accuracy for forecasting their behavior during complex cyclical loading, in particular cyclic loading where a significant creep role is played in the processes of creep, was developed. As with the majority of models, this new model has applicability limitations and the reliability of use for calculations is acceptable only for cases of small differences in loading from the proportionality.

Key words: metallic materials, cyclic loading, creep deformation, modelling, reliability

Razvit je bil model o deformaciji izotropnih kovinskih materialov s ciljem, da bi se dosegla večja natančnost pri napovedi vedenja pri kompleksni ciklični obremenitvi, še posebej v primeru pomembne vloge procesov lezenja. Kot pri večini modelov ima tudi novi model omejitve pri uporabi in zanesljivost uporabe izračunov je sprejemljiva samo za primer, ko se obremenitev malo odmika od proporcionalnosti.

Ključne besede: kovinski material, ciklična obremenitev, deformacija z lezenjem, modeliranje, zanesljivost, natančnost

1 INTRODUCTION

Several models have been developed so far [1], and some of them are in commercial use as software packages, for example, ANSYS, MARC, NASTRAN, ABAQUS, LUSAS, LS-DYNA, COSMOS, ALGOR. However, these models fail to adequately describe the case of complex cyclic loading, when creep processes also play an important role.

In analyzing the lines of access to the development of a theoretical explanation for the straining under cyclic non-isothermal loading, which is necessary for practical calculations of the strain-stressed state (SSS), the authors stood at a crossroads. It was possible to use structural and physical models, which made it possible to describe a wide range of peculiarities of a material's behaviour under complex loading using a rather small number of experimental material parameters [2]. It is worth noting that the deformation and the instantaneous plastic deformations are not separated and their interconnection is included as a property of the developed model. Analogous models have not provided sufficient reliability for a quantitative calculation since the monotonic change in the material properties differs from the experimental values. It was also clear that a modification of the traditional models for plastic flow and the different creep theories, as applied to specific loadings to achieve good accuracy with the calculation, would require a large number of basic experiments to obtain an acceptable fit for a description of real material behaviour. We chose the first solution because of its obvious advantages.

The model of deformation for isotropic metallic materials was designed to make a very accurate prediction of their behavior. This article looks at the case of complex cyclical loading.

2 THEORY

1. The variations in non-isothermal theories of plastic flow and of the theory of work hardening during creep will be included in the analysis [3], allowing us to consider the mutual effect of the two forms of deformation within the framework of the traditional approach. In rating the correctness of these proposals, we will start from the necessity of fulfilling the following requirements:

a) a description on a non-isothermal cyclic deformation;
b) a consideration of the cyclic instability of the material properties;
c) a description of the conditions of deformation for complex loading, in particular of alternating sign;
d) a consideration of the mutual influence of time-dependent and time-independent deformation.

The approach based on the separation of irreversible deformation into time-dependent and time-independent has a physical basis.

2. We will assume that the total material deformation consists of the elastic deformation r_{ij}, the creep deformations p_{ij}, and the plasticity ε_{ij}, [4], thus:

$$e_{ij} = r_{ij} + p_{ij} + \varepsilon_{ij} \tag{1}$$

In the formulation of the rules of deformation we will consider the effect of accumulated plastic deformation on the creep and of the temperature-time prehistory on the elasto-plastic properties. Although the assumption that only the second invariant of the tensor of stresses enters into the relationship for the increase of the deformation and stresses, it is only a special case of the relationship recommended for the description of the complex stressing of bodies [5]. At present this is a traditional approach. This conclusion is related to the fact that with comparatively small plastic deformations, it provides, as a rule, quite good agreement with experimental data for many cases when plastic deformation occurs during stress that is not very different from the uniaxial.

Having accepted the hypothesis of work hardening that, in particular, means the neglecting of processes of the reverse elastic secondary effect and assuming that the recurrence of loading and preliminary plastic deformation affects only the scalar properties, we may write the equation for creep rate for cyclic loading with an alternating sign:

$$p_{ij}^{(n)} = F(\overline{p}^{(0)}, \overline{p}^{(1)}, \dots, \overline{p}^{(n)}, \overline{\varepsilon}^{(0)}, \overline{\varepsilon}^{(1)}, \dots, \dots, \overline{\varepsilon}^{(n-1)}, \overline{\varepsilon}^{(n)}, \overline{\sigma}, n, T)\frac{S_{ij}}{\overline{\sigma}} \tag{2}$$

Here, S_{ij} are the components of the deviator of the stresses, $\overline{\sigma}$ is the intensity of the stresses, and $\overline{\varepsilon}^{(n)}$ and $\overline{p}^{(n)}$ are the intensities of the plastic deformation and the creep deformation determined from the equations

$$\overline{p}^{(n)} = \int_{t_{n-1}}^{t_n} \left(\frac{2}{3}\dot{p}_{ij}\dot{p}_{ij}\right)^{1/2} \mathrm{d}t \tag{3}$$

$$\overline{\varepsilon}^{(n)} = \int_{t_{n-1}}^{t_n} \left(\frac{2}{3}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}\right)^{1/2} \mathrm{d}t \tag{4}$$

where n is the number of the half cycle

$$p_{ij}\Delta p_{ij} \geq 0 \tag{5}$$

Experiments relating to uniaxial stressing [6] showed that by counting the creep deformations during cyclic loading from the start of a half cycle, the curves of the irreversible deformations accumulated during the non-steady creep are similar to the creep curves for the initial condition. In general, the coefficient of similarity depends on the number of the half cycle, the time, the amount of creep deformation accumulated per half cycle, and the temperature. The rate of steady creep is virtually independent of the number of half cycles.

Comparatively small previous creep and instantaneous plastic deformations 0.2–5.0 % may have a substantial influence on the creep rate. Plastic deformations of the opposite sign accelerate the creep of high-temperature materials of different classes (the Bauschinger effect in creep), while plastic deformations of the same sign can accelerate or retard creep, depending upon their size and the type of material.

In cases when the material is submitted to elasto-plastic steady-stage creep the deformation of the opposite sign, further creep, as a rule, starts with the non-steady stage.

An analysis of the experimental data on cyclic creep makes it possible to select two variations of the concretization of Eq. (2).

The first approach is based on the use of the graph in **Figure 1**, based on the assumption that the effect of plastic deformations on the creep rate may be considered with an appropriate change of the value of creep deformation in the relationship $\dot{p} = f(p,\sigma)$. In this case, in the plastic deformation ε_{pl} for the total deformation p, corresponding to the point k, we have a creep rate that corresponds to the point b, the point d, and not to the point b. For $p = 0$ the value of p corresponds to the point e.

The material cyclic creep instability will be considered with the use of the function φ of the number of cycles [5]. In this case

$$\dot{p} = f(\overline{\tilde{p}}, \overline{\sigma}) \tag{6a}$$

Here

$$\overline{\tilde{p}} = p + \left[1 - \tilde{\psi}_1(\overline{\varepsilon}^+)\tilde{\psi}_2(\overline{\varepsilon}^-)\varphi(n, \overline{p}^{(n-1)}\right]\overline{p}_n$$

where $\overline{p}_n$ is the intensity of the creep deformation accumulated as a result of non-steady creep; $\overline{\psi}_1$, $\overline{\psi}_2$, and ε are functions taking into consideration the effect of plastic deformations and the number of cycles and satisfying the following conditions: for $\overline{\varepsilon}^+ = 0$ $\tilde{\psi}_1 = 1$; for $\overline{\varepsilon}^- = 0$ $\tilde{\psi}_2 = 1$; for $n = 1$ $\tilde{\varphi} = 1$. $\overline{\varepsilon}^+$ and $\overline{\varepsilon}^-$ are the sum

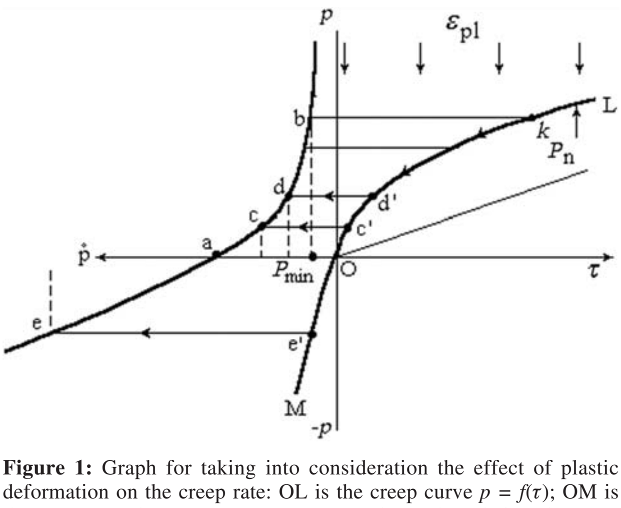

Figure 1: Graph for taking into consideration the effect of plastic deformation on the creep rate: OL is the creep curve $p = f(\tau)$; OM is the tangent to the creep curve at $\tau = 0$; a, b are the relations between the creep rate and the accumulated creep deformation; ae is the tangent to the curve ab at the point a; p_n is the creep deformation in the non-steady stage. The arrows show the method of determining the effect of ε_{pl} on $\dot{p}$.

Slika 1: Grafikon za upoštevanje vpliva plastične deformacije na hitrost lezenja. OL-krivulja lezenja $p = f(\tau)$; OM-tangenta na krivuljo lezenja v točki $\tau = 0$; a, b-odnos hitrosti lezenja in nakopičene deformacije z lezenjem; ae-tangenta na krivuljo ab v točki a; p_n-deformacija z lezenjem v nestabilnem stanju. Puščice prikazujejo metodo določitve vpliva ε_{pl} na $\dot{p}$.

of the intensities of the plastic deformations in those half cycles where $\Delta\varepsilon_{ij}^{(n)}\dot{p}_{ij}^{(n)} > 0$ and $\Delta\varepsilon_{ij}^{(n)}\dot{p}_{ij}^{(n)} < 0$;

$$\Delta\dot{\varepsilon}_{ij}^{(n)} = \int_{t_{n-1}}^{t_n} e_{ij}\,\mathrm{d}t \tag{7}$$

The problem of the use of Eq. (6a) is related to the correctness of the extrapolation of the relationship $\dot{p} = f(p,\sigma)$ in the area of negative values of p.

The second possible approach to the treatment of existing data provides, as a solution of Eq. (2), the following expression

$$\dot{\bar{p}}^{(n)} = \left[\dot{\bar{p}}_{\min} + \varphi(n,t,\bar{p}^{(n-1)}(p_0 - p_{\min})\right] \tag{6b}$$

where $\dot{\bar{p}}_{\min}$ is the steady-stage creep rate; $\dot{\bar{p}}_{\min} = f(\bar{\sigma},T)$; $\dot{\bar{p}}_0$ is the initial creep rate of the material, calculated according to the theory of work hardening; $\dot{\bar{p}}_0 = f(\bar{\sigma},T,\bar{p})$; $\varphi(n,t,\bar{p}^{(n-1)})$ is a function considering the effect of the number of cycles on the non-steady creep; $\psi_1(\bar{\varepsilon}^+,\tau)$ and $\psi_2(\bar{\varepsilon}^-,\tau)$ are functions considering the effect of plastic deformations; t and τ are the times counted from the start of the cycle and the moment of the start of the plastic deformation.

The form of the functions $\dot{\bar{p}}_{\min}$ and $\dot{\bar{p}}_0$ may be determined in creep tests under conditions of the uniaxial stressed condition at constant values of the stresses and the temperature.

In Equations (2) and (6) the effect of the temperature at which the instantaneous plastic deformation was accumulated on creep rate is neglected. Such an effect is possible; however, the existing experimental results indicate that it is insignificant.

3. We will describe the instantaneous plastic deformations for non-isothermal cyclic deformation of alternating sign with the incremental theory of thermoplasticity with a **piecewise-spherical** surface [3], modified by applying a relationship for the accumulated creep deformation. Let us assume that the original material is isotropic in the space of the deviators of stresses and it has the paths of the cyclic load for each point of the body given by the cone with a small spatial angle α (**Figure 2**). The area where the vector deviator of stresses during the whole load cycle must be found is cross-hatched.

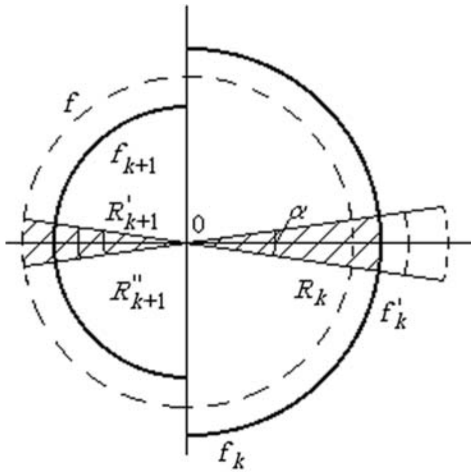

Figure 2: Graph of the surface of flow during cyclic loading: f) original surface of flow; f_k, f_{k+1}) parts of the surface after the k-th and the (k+1)-th half cycles; R'_{k+1} and R''_{k+1}) radius of the surface of flow in the (k+1)-th half cycle with the related increase of stresses in the k-th half cycle.

Slika 2: Grafikon za površino lezenja pri ciklični obremenitvi: f) izvirna površina lezenja; f_k, f_{k+1}) deli površine po k in (k+1) polovičnem ciklu; R'_{k+1} in R''_{k+1}) polmer površine lezenja pri (k+1) polovičnem ciklu z od njega odvisnim povečanjem napetosti pri k polovičnem ciklu

For the k-th half cycle we have

$$f_k = \sqrt{S_{ij}S_{ij}} - r(\bar{\varepsilon}^{(0)}, \bar{\varepsilon}^{(1)}, \ldots, \bar{\varepsilon}^{(k-1)}, \bar{\varepsilon}^{(k)}, \bar{p}^{(0)}, \bar{p}^{(1)}, \ldots, \bar{p}^{(k)}, k, T) = 0 \tag{8}$$

where the plastic work hardening does not depend on the temperature prehistory. As for creep, the number of the half cycle is increased per unit with a breakdown in the condition

$$\varepsilon_{ij}\Delta\varepsilon_{ij} \geq 0 \tag{9}$$

where

$$\Delta\varepsilon_{ij} = \sqrt{\frac{2}{3}}\Delta\varepsilon\frac{S_{ij}}{R} \tag{10}$$

$\Delta\bar{\varepsilon}$ is the intensity of the increase in deformations.

A determination of the radius of the surface of flow R is significantly easier, applying for the material and temperature T, the generalized diagrams of cyclic deformation representing the relationships between the increases in stresses and deformations counted from the start of a given half cycle and independent of the amplitude of the cycle deformation.

If we assume that between the increase of stresses and the accumulation of plastic deformation in a given half cycle at a given temperature and constant values of $\bar{p}$ a single relationship exists, we can determine the value of the radius of the surface of flow as an algebraic sum of its value in the zero half cycle and its increase in the subsequent half cycles:

$$R_k = R_0 + \sum_{i=1}^{k}\Delta R_i \tag{11}$$

The treatment of the results of the experiments made for uniaxial stressing showed that a small creep deformation increases the yield strength of a material in cases when the directions of deformation coincide (**Figure 3**), and decrease it when the directions of deformation in creep and in instantaneous elastoplastic deformation are opposite.

We can assume that the creep deformation $\bar{p}$ influences the value of the radius of the surface of flow by an additional plastic deformation of $\bar{\bar{\varepsilon}} = (\bar{p}/\varepsilon_c)\varepsilon_t$, where ε_c and ε_t are the deformation capacity of the material for creep and short-term tension. In this case, the expression for the surface of flow is written in the form:

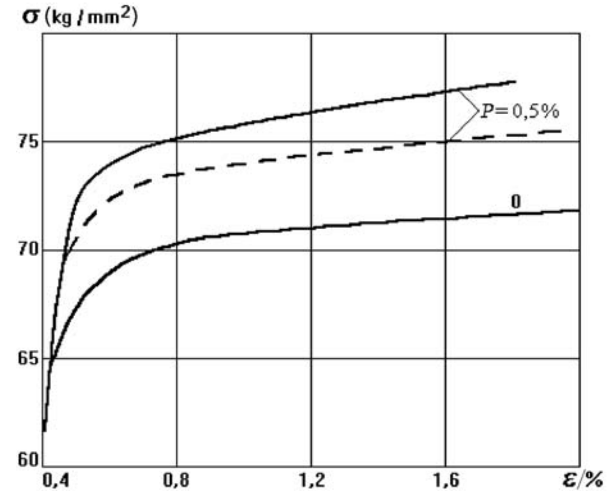

Figure 3: Effect of creep on the resistance of 25Kh2M1F steel to instantaneous deformation at 550 °C. Full lines are the experimental results; dashed line is the calculated.

Slika 3: Vpliv lezenja na odpornost jekla 25Kh2M1F proti hipni deformaciji pri 550 °C, polne črte – eksperimentalne meritve; črtkane črte – izračunano

$$f_k = \sqrt{S_{ij}S_{ij}} - r\left(\bar{\varepsilon}^{(0)} + \frac{\bar{p}}{\varepsilon_c}\varepsilon_t, \bar{\varepsilon}^{(1)} + \frac{\bar{p}^{(1)}}{\varepsilon_c}\varepsilon_t, \ldots \right.$$
$$\left. \ldots, \bar{\varepsilon}^{(k)} + \frac{\bar{p}^{k}}{\varepsilon_c}\varepsilon_t, k, T\right) = 0 \qquad (12)$$

A comparison of the calculated curve (the dashed line in **Figure 3**) with the experimental (the solid lines) confirms the acceptability of this assumption. (The calculated curve for p = 0,5 % was obtained from the curve for p = 0 by replacement to the left by the value $\varepsilon = (p/\varepsilon_c)\varepsilon_t$.)

This form of surface flow makes it possible to explain, in particular, the phenomenon of plastic deformations for a creep test cycle with alternating sign and stresses lower than the elastic limit of the original material [6].

Equations (1), (6), and (12), the expanded equations of the equilibrium and the consistency of the deformation of uniform medium and also the necessary boundary conditions make it possible to calculate the stressed-strained condition of a body in an arbitrary program of cyclic loading and heating by a step method. At the same time, Eq. (6) and (12) satisfy the requirements given earlier.

It was shown in [7] that the calculation for stresses and deformations during the loading stage may be developed as a solution to the problem of the deformation theory of plasticity with varying the parameters of elasticity [4]. The process of determining successive approximations for the stresses and deformations is carried out with the separation of the deformation into elastic and an increase in the instantaneous plastic deformation and creep according to Eqs. (6) and (12) using the method of successive approximations.

As a parameter of change of materials properties related to its cyclic instability by a change of cyclic loading in place of the number of semi-cycles, the path of cyclic creep, λ_1, and the path of cyclic plastic deformation λ_2, should be accepted. Let us assume that the cyclic creep and its increment are determined by the equations:

$$\begin{aligned} \lambda_1 &= \int \mathrm{d}\varepsilon_p - \varepsilon_p ; \mathrm{d}\varepsilon_p = (2/3 \cdot \mathrm{d}\varepsilon_{pij} \cdot \mathrm{d}\varepsilon_{pij})^{0.5} ; \\ \varepsilon_p &= (2/3 \cdot \varepsilon_{pij} \cdot \varepsilon_{pij})^{0.5} \\ \lambda_1 &= \int \mathrm{d}p - p; \mathrm{d}p = (2/3 \cdot \mathrm{d}p_{pij} \cdot \mathrm{d}p_{pij})^{0.5} ; \\ p &= (2/3 \cdot p_{pij} \cdot p_{pij})^{0.5} \end{aligned} \qquad (13)$$

$$\Delta\lambda_i = \lambda_i^{(k)} - \lambda_i^{(k-1)} \geq 0 \qquad (14)$$

Here, the following requirements must be met:

$$\frac{\partial\lambda_i^{(k)}}{\partial\tau} > 0 \text{ at } t < t_k \ ; \ \frac{\partial\lambda_i^{(k)}}{\partial\tau} = 0 \text{ at } t > t_k \qquad (15)$$

By satisfying conditions (14) and (15), the value of n increases by unity.

The increments of the inelastic deformation and the values of the intensity of the inelastic deformation are determined from the equations:

$$\mathrm{d}\varepsilon_{ij}^{\aleph} = \mathrm{d}\varepsilon_{pij} + \mathrm{d}p_{ij} ; \ \varepsilon^{\aleph} = (2/3 \cdot \varepsilon_{ij}^{\aleph} \cdot \varepsilon_{ij}^{\aleph})^{0.5}$$

4. An estimation procedure for the material characteristics and design procedure of creep curves for some typical examples of uniaxial loading with cyclic creep at varying temperatures can be applied. The proposed method can be used to calculate the strains at all three stages of creep for heat-resistant steels and alloys with an arbitrary law of change in the stress and temperature at the working temperatures at which the material is structurally stable. The method cannot be used for calculations at the third stage of creep in materials that are fractured after necking or in cases of compression or alternating loading of materials with a highly anisotropic initial creep resistance (in tension and compression).

The duration of the creep process during one cycle may range from a minute to hundreds of hours. In developing the method, we analyzed data on creep and stress relaxation in 20 grades of heat-resistant steels and alloys in a uniaxial stress state.

3 COVERNING EQUATIONS

The method is based on a creep theory of the following type [8]

$$\dot{p} = f(\sigma, p, T, \varepsilon_{\mathrm{pl}}, \lambda_1) \qquad (17)$$

where $\lambda = \int (|\mathrm{d}p| - \mathrm{d}p)$ is the path of cyclic creep.

Tests involving a single loading are approximated with the following analytical formula:

$$p = F(\sigma, T, \varepsilon_{\mathrm{pl}}, t) \qquad (18)$$

The relation for creep rate in the same tests can be determined by differentiating Eq. (18):

$$\dot{p} = \frac{\mathrm{d}F}{\mathrm{d}t} = \Phi(\sigma, T, \varepsilon_{\mathrm{pl}}, t) \tag{19}$$

The behaviour of the material by complex loading programs is assumed to be described by creep theory (17); thus, it is obvious that the required relationship $p = f(\sigma, p, T, \varepsilon_{\mathrm{pl}}, \lambda)$ should be obtained by excluding t from Eqs. (18) and (19). However, this cannot be done analytically in a general form with the present form of Eq. (19); it is therefore proposed that in the solution of any specific problem, the value of function [17] should be found numerically by excluding t from Eqs. (18) and (19) at each step of the integration over time. For alternating loading and the absence of instantaneous plastic strains, the specific form of Eq. (18) to describe the creep curves in the first and second stages is:

$$p = A\sigma^k \left[1 - \exp(-C\sigma^l t)\right] + B\sigma^m t \tag{20}$$

where A,B,C,k,l,m are coefficients that are constant for a given test temperature. To describe the third creep stage, we replace the stresses in Eq. (20) with the ratio $\sigma/(l - p/\varepsilon_f)$, where ε_f is the strain at failure.

The change of the creep curve is, for the case of plastic deformation, accounted for by replacing (20) with the expression:

$$p = A\sigma^k \left\{1 - \exp\left[(-C\sigma^l (t - t_a)\right]\right\} S(\varepsilon_{\mathrm{pl}}) + B\sigma^m (t - t_a) \tag{21}$$

where t_{a} is the time to the last plastic deformation. Here, we have in mind, not creep that is accompanied by a continuous change of instantaneous plastic strain, but the effect of discrete instantaneous plastic strain at the moment of the application of the plastic deformation $\varepsilon_{\mathrm{pl}}$.

When the plastic strain $\varepsilon_{\mathrm{pl}}$ is accumulated under stress in creep tests, the effect of $\varepsilon_{\mathrm{pl}}$ is automatically accounted for with the coefficients A,B,C,k,l,m.

The values of the function S, describing the effect of plastic strain on creep, depend on the sign of $\varepsilon_{\mathrm{pl}}$. The form of the function $S(\varepsilon_{\mathrm{pl}})$ describing the effect of plastic strain on creep rate, dependent on the sign of $\varepsilon_{\mathrm{pl}}$, is determined from a series of tests with different values of $\varepsilon_{\mathrm{pl}}$ and is either specified exactly or given by the approximating function:

$$S(\varepsilon_{\mathrm{pl}}) = 1 + h\varepsilon_{\mathrm{pl}}^q \tag{22}$$

It has been established that the values of the material parameters h and q are slightly dependent on the stress level. The dependence of $\dot{p}$ on the sign of the stresses is taken into account as follows:

$$\dot{p} = f(|\sigma|)\,\mathrm{sign}\,\sigma \tag{23}$$

The effect of cyclic loading on the creep rate is considered with inclusion of the function $f_1(\lambda)$ in Eq. (19) as a multiplier:

$$\begin{aligned} p = A\sigma^k &\left\{1 - \exp\left[(-C\sigma^l (t - t_a)\right]\right\} \cdot \\ &\cdot S(\varepsilon_{\mathrm{pl}}) f_1(\lambda) + B\sigma^m (t - t_a) \end{aligned} \tag{24}$$

where t_{a} is the time to the last plastic deformation or to the last change in the stress sign.

The function $f_1(\lambda)$ is determined from tests in cyclic loading with constant cycle parameters: under these conditions, $f_1(\lambda) = f_2(k)$, where k is the number of the cycle. For k = 1, $f_2(k) = f_1(\lambda) = 1$.

Thus, with the chosen test temperatures, T_i (i = 1 ... N), we have the following expressions for p and $\dot{p}$:

$$\dot{p} = \left\{ A_i C_i \left(\frac{|\sigma|}{1 - \frac{|p|}{\varepsilon_{\mathrm{fi}}}} \right)^{k_i + l_i} \exp\left[-C_i \left(\frac{|\sigma|}{1 - \frac{|p|}{\varepsilon_{\mathrm{fi}}}} \right)^{l_i} (t - t_a) \right] S(\varepsilon_{\mathrm{pl}}, \mathrm{sign}(\varepsilon_{\mathrm{pl}}, \sigma)) f_1(\lambda) + B_1 \left(\frac{|\sigma|}{1 - \frac{|p|}{\varepsilon_{\mathrm{fi}}}} \right)^{m_i} \right\} \mathrm{sign}\,\sigma \tag{25}$$

$$p = \left\{ A_i \left(\frac{|\sigma|}{1 - \frac{|p|}{\varepsilon_{\mathrm{fi}}}} \right)^{k_i} \left\{ 1 - \exp\left[-C_i \left(\frac{|\sigma|}{1 - \frac{|p|}{\varepsilon_{\mathrm{fi}}}} \right)^{l_i} (t - t_a) \right]\right\} S(\varepsilon_{\mathrm{pl}}, \mathrm{sign}(\varepsilon_{\mathrm{pl}}, \sigma)) f_1(\lambda) + B_1 \left(\frac{|\sigma|}{1 - \frac{|p|}{\varepsilon_{\mathrm{fi}}}} \right)^{m_i} \right\} \mathrm{sign}\,\sigma \tag{26}$$

To find $\dot{p}$ with a known value of p at each step of the integration in accordance with (17), it is necessary to exclude the parameter $t - t_{\mathrm{a}}$ from these equations.

The value of $\dot{p}$ with an arbitrary temperature T is determined by interpolating the values of $\dot{p}$, found using the above-described method with several of the nearest values of T_i.

As an additional material parameter we consider the limiting temperature T_0: at $\dot{T} \le T_0$, $p = 0$.

If the calculations are performed with a change of sign for the stress σ at any number of times (i.e., when the continuous function σ passes through zero), the value of $\dot{p}$ changes as follows.

Instead of (17) we have

$$\dot{p} = f(\sigma, p - a, T, \varepsilon_{\mathrm{pl}}, \lambda_1) \tag{17a}$$

where a is the creep accumulated up to the moment of the change in the sign of the stresses.

We should additionally assume that for a complex stress state the cyclic loading and the preliminary plastic deformation affects only the scalar properties of the materials. Then, instead of (17) we can write [9]

$$\begin{aligned} \dot{p}_{ij}^{(n)} = F(&\bar{p}^{(0)}, \bar{p}^{(1)}, \ldots, \bar{p}^{(n-1)}, \bar{p}^{(n)}, \bar{\varepsilon}^{(0)}, \bar{\varepsilon}^{(1)}, \ldots, \\ &\ldots, \bar{\varepsilon}^{(n-1)}, \bar{\varepsilon}^{(n)}, \bar{\sigma}, \lambda, T) \frac{S_{ij}}{\bar{\sigma}} \end{aligned} \tag{27}$$

Here, S_{ij} are the components of the stress deviator; $\bar{\sigma}$ is the stress intensity; $\bar{p}^{(n)}$ and $\bar{\varepsilon}^{(n)}$ are the intensities of the creep strain and (non-creep) plastic strain:

$$\bar{p}^{(n)} = \int_{t_{n-1}}^{t_n} \left(\frac{2}{3} \dot{p}_{ij} \dot{p}_{ij} \right)^{1/2} \mathrm{d}\tau;\ \bar{\varepsilon}^{(n)} = \int_{t_{n-1}}^{t_n} \left(\frac{2}{3} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij} \right)^{1/2} \mathrm{d}\tau \tag{28}$$

where n is the number of half-cycles.

4 METHOD OF DETERMINING THE MATERIAL'S CHARACTERISTICS

The system of equations (18–20) contains several material constants

$$\begin{array}{ccccccccc} A_1 & B_1 & C_1 & k_1 & l_1 & m_1 & \varepsilon_{p1}(t) & h_1 & q_1 \\ A_2 & B_2 & C_2 & k_2 & l_2 & m_2 & \varepsilon_{p2}(t) & h_2 & q_2 \\ . & . & . & . & . & . & . & . & . \\ A_N & B_N & C_N & k_N & l_N & m_N & \varepsilon_{pN}(t) & h_N & q_N \end{array}$$

T_0 and the functions $S[\varepsilon_{pl}, \mathrm{sign}(\varepsilon_{pl},\sigma)]$, $f_1(\lambda)$.

The constants A_i,B_i,C_i,k_i,l_i,m_i are determined from an analysis of the creep data for uniaxial stress in the first and second stages at N temperatures T_i and several (at least six) stresses.

We select three numbers, a, b and c, determining the first and second creep stages for each experimental creep curve (**Figure 4**).

The data on a, b and c for different values of σ and T = const are used to find the values of A, B, C, k, l, m, applying the method of least squares and the basis of the following power relations:

$$a = A\sigma^k ; b = B\sigma^k ; c = C\sigma^l \tag{29}$$

The parameters h_i and q_i of the function $S(\varepsilon_{pl})$ are determined with the analysis of the results of creep tests at n temperatures T_i, one stress for each value of T_i, and several values of preliminary plastic strain ε_{pl} within the range from –3 % to –5 % to +3 % to + 5 % (at least three values $\varepsilon_{pl} < 0$, three values $\varepsilon_{pl} > 0$, and $\varepsilon_{pl} = 0$).

For materials with the function $S(\varepsilon_{pl})$ depending on the stress level, it is better if the results of the determination of $S(\varepsilon_{pl})$ are obtained using exact specifications for the values of $S(\varepsilon_{pl},\sigma)$.

The function $f_1(\lambda)$ is found with the analysis of results of the creep tests with alternating loading in the program shown in **Figure 5** and the determination of the dependence on the cycle (the creep strain accumulated within a half-cycle) **Figure 6**.

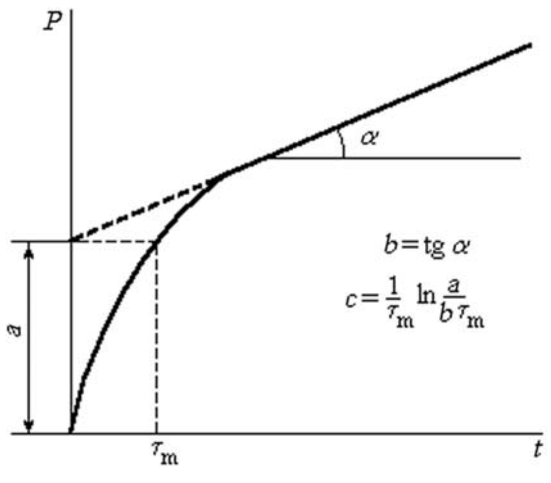

Figure 4: Creep curve
Slika 4: Krivulja lezenja

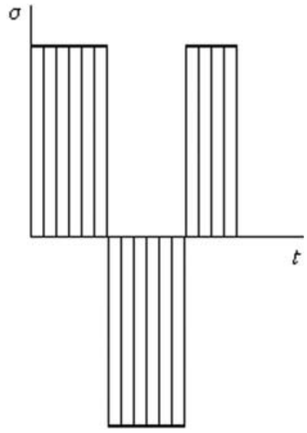

Figure 5: Diagram of loading of the specimens
Slika 5: Diagram obremenitve preizkušancev

5 EXAMPLES OF THE CALCULATION

The values of the coefficients in Eqs. (25) and (26) for the alloy KhN70VMYuT (EI765) are shown in **Table 1**.

The creep strain that occurs with an arbitrary law of change of stress, temperature, and instantaneous plastic strain is determined by the numerical integration of Eqs. (17) and (17a) using the fourth-order Runge-Kutt method and a computer algorithm. The algorithm can be obtained without a computer by using a simple law of

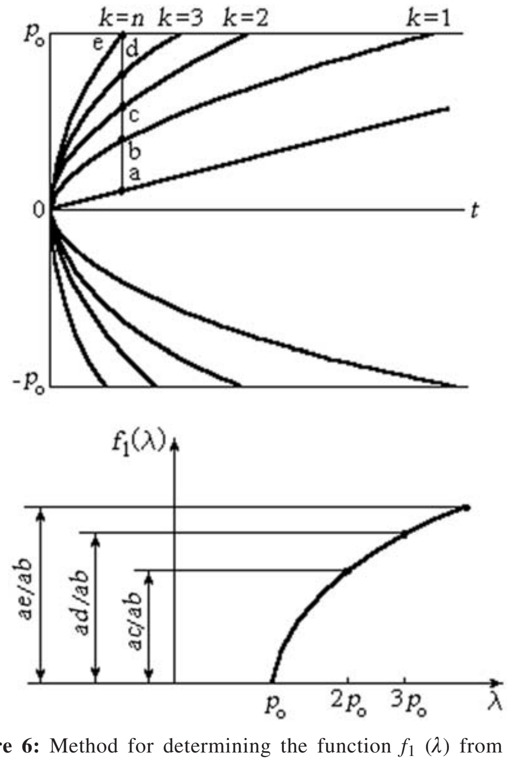

Figure 6: Method for determining the function f_1 (λ) from cyclic creep curves.
Slika 6: Metoda za določitev funkcije f_1 (λ) iz krivulje cikličnega lezenja

Table 1: Parameters of the creep resistance of the alloy EI765
Tabela 1: Parametri odpornosti proti lezenju za zlitino E1765

Temp., T/°C	ε_n/%	ε_p/%	h_1	l_1	$f_2(15)$	σ_y/ (kg/mm^2)	Stress level	–lg A	–lg B	–lg C	k_0	l_0	m
650	8	17	–	–	–	–	$\sigma < \sigma_y$	22,3	37,9	12,96	10,3	7,63	18,75
700	8-14	14	26	0,5	–	64,5	$\sigma < \sigma_y$	13,62	16,3	9	5,56	5,92	7,41
							$\sigma > \sigma_y$	27,7	39,2	9	13,33	5,92	20
750	13-15	16	350	1,3	4	55,6	$\sigma < \sigma_y$	5,8	16,62	2,8	1,14	2,68	8,33
							$\sigma > \sigma_y$	18,4	28,9	2,8	8,33	2,68	15,38

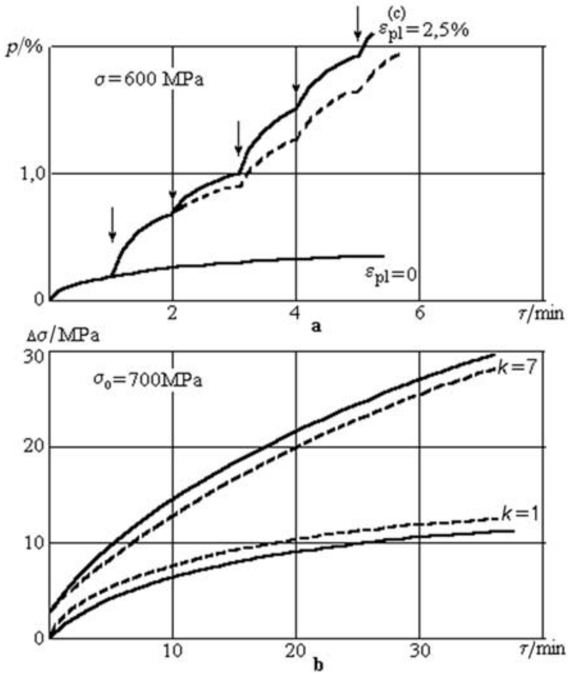

Figure 7: Experimental (full lines) and theoretical (dashed lines) curves describing the effect of cyclic plastic deformation on the creep resistance (a) and the relaxation (b) for the alloy EI765 at 700 °C
Slika 7: Eksperimentalne (cele črte) in teoretične (črtkane črte) krivulje, ki opisujejo vpliv ciklične plastične deformacije na odpornost proti lezenju (a) in relaksacijo (b) pri 700 °C za zlitino EI765

the change in stress $\sigma(t)$, temperature $T(t)$, and plastic strain $\varepsilon_{pl}(t)$.

As examples of the calculation, the following variants were analysed [6,10]: creep during the first and third stages (σ = const, T = const); for a temperature change (σ = const); for an alternating stress (T = const); for conditions of cyclic plastic deformation (σ = const, T = const); for cyclic creep with alternating plastic strain; with plastic strain and a changing stress (T = const); for alternating stress during a changing temperature. Also, the stress relaxation with $\sigma_0 < \sigma_y$; with $\sigma_0 > \sigma_y$ and additional preliminary plastic deformation; with additional loadings and cyclic plastic strain were examined.

The analysis of the agreement for theoretical and experimental data for twenty different grades of steels and alloys showed that the proposed creep model and the method of determining its parameters were valid (see **Figure 7**, for example).

The studies [11,12] proposed variants of the above method for calculating the creep to determine the stress-strain state of blades (uniaxial stress state) and disks (complex stress state) with multiple starts of gas-turbine engines.

6 CONCLUSION

The variations in aniso-thermal theories of plastic flow and of the theory of work hardening in creep with structural parameters have been considered, making it possible to include the mutual effect of two forms of deformations within the framework of the traditional approach.

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