A MATHEMATICAL MODEL FOR THE STATIONARY PROCESS OF ROLLING OF TUBES ON A CONTINUOUS MILL

MATHEMATICAL MODEL PROCESA KONTINUIRNEGA VALJANJA CEVI

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1 INTRODUCTION

The prospects of enhancement of the production efficiency at numerous tube rolling units are closely linked with the possibility of a reliable prediction of the forming parameters at the final stage of plastic deformation in the plugless tube reducing or sizing processes. In this connection, the problem of development of a universal mathematical model applicable in studying the process of lengthwise plugless rolling in the tube rolling mills equipped with the roll drives of different types is of a high interest.

2 STATE OF THE ISSUE AND THE AIM OF INVESTIGATION

The analysis of the relevant references shows that the problem of determination of kinematical, deformatonal and power-and-force parameters of the continuous plugless lengthwise tube rolling process was solved up to now by consecutive analysis of forming in each individual stand. Solutions based on integration of the deformational parameters in all N stands of the continuous mill into a common system of equations are proposed, also in 1,2. In the development of mathematical models of the continuous rolling process, e.g. in 1,2 two assumptions were made.

Firstly, the mean angle of the neutral section \( \theta_{n} \) (Figure 1) is defined for the condition of coincidence of the roll and the mother tube speeds within the section of the deformation zone exit in the \( i \)-th stand, though it would be logical to choose some section between the entry and exit of the deformation zone.

Secondly, for the determination of the effective roll diameter \( D_{e} \), the approximate formula is used: \( D_{e} = D_{i} - D_{i} \cos \theta_{i} \) (where \( D_{i}, D_{i} \) are the ideal roll diameter and the mean tube diameter after rolling in the \( i \)-th stand respectively) that introduces an error because in reality \( D_{e} = D_{i} - 2r_{n} (\theta_{n}) \cos \theta_{i} \) (where \( D_{i} \) is the ideal roll diameter; \( r_{n} (\theta_{n}) \) is the pass radius at \( \theta = \theta_{n} \), see Figure 2).

Figure 1: Scheme for the determination of the value of the effective diameter \( D_{k} \)

Slika 1: Shema za določitev efektivnega premera \( D_{k} \)
In accord with the model consisting of $2N$ equations proposed\(^1\), the effective roll diameter $D_{hi}$ cannot be greater than the ideal roll diameter $D_0$ and smaller than the roll diameter at the swell $D_s$. This distorts the real picture of the rolling process kinematics, namely, when the rolls slip on the mother tube surface two conditions are met: $D_0 < D_{hi}$ or $D_{hi} > D_s$. The model proposed\(^2\) is free from this shortcoming but it is a system of $3N$ equations that when being solved at $N > 16$ is connected with considerable difficulties because of the great number of unknowns to be determined.

This work is aimed at the verification and simplification of the mathematical models proposed\(^1,2\) and to the assessment of the verification results on the basis of comparison of calculated and experimental data and it is, for this reason, of scientific and practical interest.

3 PROBLEM STATEMENT

The following values have to be calculated:

- angular roll rotation velocity $n_{Bi}$ in each $i$-th mill stand (for the mill with individual roll drives);
- angular velocities of rotation of the main ($N_1$) and auxiliary ($N_B$) motors (for the mill with differential-group roll drives);
- ideal roll diameters $D_{ui}$ (for the mill with group roll drives).

These values ensure that tubes of required size ($D_iS_t$ mm) are rolled from the mother tube of given size (($D_0 \times S_0$) mm) at a specified rolling speed $V_0$(m/s) in the first stand of the multiple-stand mill.

Initial data for the calculation are as follows:

- the total diameter and wall reduction (or just diameter reduction), i.e. initial mother tube dimensions $D_0 \times S_0$ (or just $D_0$) and final tube dimensions $D_i \times S_i$;
- the distribution of partial mother tube diameter reductions $m_i$ (%) among the mill stands of total number of $N$;
- the value of external friction $f_i$;
- the mother tube rolling speed $V_0$(m/s) in the first mill stand (the problem can also be stated for $V_0$ as the value to be determined);
- the gear ratios $\eta_1$, $\eta_B$ from the motors to the rolls in the lines of the main and auxiliary drives (for the mills with differential-group roll drives);
- the absence of backward pull in the first mill stand ($Z_{i1} = 0$) and of front pull in the last mill stand ($Z_{iN} = 0$);
- the number of rolls $N_k$ forming passes in the mill stands.

4 PHYSICAL MODEL OF THE PROCESS

No mother tube forming occurs in interstand spaces and the wall thickness $S_j$ at the exit from the stand of ordinal number $j = i - 1$ is equal to the wall thickness $S_{0j}$ at the entry to the stand of ordinal number $i$. The deformation resistance $K_0$ of the mother tube material at the exit from the stand of ordinal number $j$ is equal to the deformation resistance $K_{0ji}$ of the mother tube material at the entry to the stand of ordinal number $i$. It follows that the coefficient of front plastic pull $Z_{ij}$ for the stand of ordinal number $j$ is equal to coefficient of backward plastic push $Z_{ij}$ for the stand of ordinal number $i$. The area $F_{ij}$ of the contact surface of the mother tube with one roll in the stand of ordinal number $i$ is equal to the area of a rectangle with sides

$$L_i = \sqrt{\frac{\beta_i \cdot D_i \cdot \varepsilon_i \cdot (D_{ui} - D_i)}{2 \sin \beta_i}}$$

with

$$\beta_i = \frac{1}{D_i}$$

where $D_i$ and $D_j$ are the mean mother tube diameters at the entry to and at the exit from the deformation zone in the stand of ordinal number $i$:

$$\varepsilon_i, / \% = \frac{m_i}{100} \quad \beta_i = \frac{\pi}{N_w}$$

The area $F_{ij}$ of the zone of forward creep at the surface of contact between one roll and the mother tube in the deformation zone of the stand of ordinal number $i$ is defined as the surface of a rectangle with sides

$$L_i^* = L_i \quad \beta_i$$

and

$$L_j^* = \theta_{ij} D_j$$

where $\theta_{ij}$ is the neutral section angle characterizing the position of the neutral line differentiating the zone of forward creep and the zone of backward creep on the surface of contact between the mother tube and the roll in the deformation zone (Figure 1).

In a real process, the magnitude of angle $\theta_{ij}$ is a function of the angle $\alpha$ characterizing the position of a concrete diametrical section of the deformation zone relative to the diametrical section of the mother tube exit from the reduction zone. In accord with the assumption\(^4\), the magnitude of angle $\theta_{ij}$ is assumed to be equal to some quantity averaged over the contact surface length. It will be regarded that $\theta_{ij}$ is the value of the neutral angle in the "neutral" diametrical section of the deformation zone where the extension is equal to the mean extension in the $i$-th stand. The axial velocity $V_{Ai}$ of metal and axial component of the roll surface velocity $V_{Bi}$ in the "neutral" diametrical section are given with

$$V_{Smh} = V_0 \frac{\mu_{Li}^p}{\xi}$$

$$V_{mih} = \frac{\pi n_{Bi} A_{Li}^p D_{mi}}{\xi}$$

where $\mu_{Li}^p = \frac{2 \pi \cdot 5 \cdot (D_i - S_i)}{5 \cdot (D_i - S_i) + 5 \cdot (D_i - S_i)}$ is the total elongation from the mill entry to the "neutral" diametrical section of the $i$-th stand;
\( A_0^\circ \) is the mean value of the guiding cosine of the contact friction stresses \(^1\):
\[
D_0 = D_{\text{al}} - 2r_0 \cos \theta \text{ is the varying of the roll pass diameter across the pass perimeter (Figure 2)};
\]
\[
r_0 = \overline{OA} = R_0 \left[ \frac{1}{1 - \frac{e_0}{R_0}} \left( 1 - \cos^2 \theta \right) - \frac{e_0 \cos \theta}{R_0} \right]
\]
\( D_0 = \overline{OA} = \frac{h_0 (\lambda_0^2 + 1 - 2 \lambda_0 \sin \psi)}{2(1 - \lambda_0 \sin \psi)} \) is the pass generatrix radius;
\( e_0 = \overline{OA} = \frac{h_0 (\lambda_0^2 - 1)}{2(1 - \lambda_0 \sin \psi)} \) is the pass generatrix eccentricity;
\( \lambda_0 = \frac{b_0}{h_0} \) is the pass ovality;
\( \psi = \frac{(N - 2)}{2N_b} \) is the pass shape index;
\( b_0, h_0 \) are the pass width and the pass height correspondingly;
\( \xi = 6 \times 10^4 \) is coefficient of quantity dimension reduction (s-mm\(^{-1}\)-min\(^{-1}\)).

The angle \( \theta_0 \) is defined as root of the transcendental equation
\[
V_{\text{Min}} - V_{\text{BMin}} = 0 \tag{7}
\]
Taking in account that in a physical sense \( 0 \leq \theta_0 \leq \beta_0 \), the condition for the determination of the neutral angle assumes the following form (in symbols of MathCAD programming language)
\[
\theta_0 = \begin{cases} 
\arccos Q_i \text{ if } \cos \beta_i \leq Q_i \leq 1 \\
0 \text{ if } 1 < Q_i \\
\beta_i \text{ if } \cos \beta_i > Q_i
\end{cases} \tag{8}
\]

The quantity \( Q_i \) in (8) is defined as the root of equation
\[
\frac{2S_i V_i (D_i - S_i)}{S_i (D_i - S_i) + S_i (D_i - S_i)} - \frac{\pi r_i A_i^\circ}{\xi} = 0 \tag{9}
\]

Imagine: \( D_i \) is the varying of the roll pass diameter, \( A_i^\circ \) is the varying of the roll pass diameter, \( \xi = 6 \times 10^4 \) is coefficient accounting for the effect of the contact friction stresses upon normal contact stresses; \( m_i = 1 + 0.36 \frac{f_i}{N_b} \) is coefficient accounting for the effect of the contact friction stresses upon normal contact stresses; \( Z_{ni} \) is coefficient of forward plastic pull; \( Z_{qi} = \frac{Z_{ni} + 2Z_{ni}}{3} \) is the mean value of the plastic pull coefficient in the i-th stand.
Table 1: Parameters of rolling a (57 × 11.6) mm tube from a (117 × 14.8) mm mother tube

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<th>i</th>
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<th>m%</th>
<th>λi</th>
<th>S/mm</th>
<th>Zui</th>
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NOTES:
A = calculation by the procedure proposed in 1
B = calculation by the procedure proposed in this work
E = experimental data

5 MATHEMATICAL MODEL

Equate right parts of equations (8) and (11) and use the equation of relation between the change of the mean wall thickness and force conditions of the mother tube deformation in each i-th mill stand to obtain the mathematical model of the continuous mother tube rolling process in N stands of the mill as a system of 2N equations:

\[
\theta_{ui} = \begin{cases} 
\frac{\sin Q_i}{\cos \beta_i}, & \text{if } \cos \beta_i \leq Q_i, \leq 1 \\
\frac{1 + \sin Q_i}{\cos \beta_i}, & \text{if } 0 \leq \sin \beta_i, \leq 1 \\
\frac{1}{\cos \beta_i}, & \text{if } 0 \leq X_i, \leq 1 \\
\frac{1 + \sin \beta_i}{\cos \beta_i}, & \text{if } 1 < X_i \\
\end{cases}
\]

where

\[
\varphi_i = \ln \left( \frac{D_i - S}{D_i - S_i} \right); T_i = \left( \frac{S_i + S_i}{D_i} \right)^k; \]

(\(Z_{\varphi} \)) = \frac{1}{2} (\(Z_u \) + \(Z_u \));

K = 1.57 for \(N_0 = 2; K = 1.20 \) for \(N_0 = 3; i = 1, 2, \ldots, N-1, N \)

Distinct from the known solution\(^2\), the mathematical model includes 2N and not 3N equations that simplifies the search of solution and makes it possible to analyze the rolling process in stretch-reducing mills with \(N \leq 25 \) stands.

Depending on the type of the mill drive, the problem of determination of the rolling parameters with the use of the system of equations (12)-(13) can be formulated in different ways. For the mills with individual drives, it is necessary to determine 2N values of \(\eta_{ui} \) (where \(i = 1, 2, \ldots, N \) and \(S_i \) (where \(i = 0, 1, 2, \ldots, N-1 \) for the specified values of \(S_i, V_0 \) and \(Z_{ui} \) (where \(i = 1, 2, \ldots, N-1 \) for the specified values of \(S_i, V_0 \)). For the mills with differential-group drives, it is necessary to find 2(N-1) values of the quantities \(S_i \) and \(Z_{ui} \) (where \(i = 1, 2, \ldots, N-1 \) for the specified values of \(S_i, V_0 \)).
of the quantities $D_u$ (where $i = 2, 3, \ldots, N$), $N$ values of the quantities $S_i$ ($i = 0, 1, 2, \ldots, N-1$) and the angular roll velocity $n_0$ that is constant for all stands at the specified values of $D_u$, $V_0$ and $Z_0$ (where $i = 1, 2, \ldots, N-1$).

Solve the system of equations (12)-(13) using (8) or (11) to find values of neutral angles $\theta_n$ and determine the values of the effective diameters in correspondence with expression (10).

6 RESULTS OF MODEL CALCULATIONS

The model has been successfully used in the calculation of tube rolling parameters for mills with the roll drives of individual, group and differential-group types.

As an example, let us consider the results of the calculation of the nature of change in the mean wall thickness $S$, rolling pressure $P$, rolling moments acting in the stand $M_1$ and the values of $Z_{al}$, $\theta_{al}$ in rolling a $D_1 \cdot S_1 = (57 \times 11.6)$ mm tube from a $D_0 \cdot S_0 = (117 \times 14.8)$ mm mother tube in 23 stands of the tube rolling unit "30-102" reducing mill with differential-group roll drives ($N_B = 3$, $V_0 = 0.7$ m/s with the mother tube material: Grade 45 steel). In this case, the mathematical model (12)-(13) is a system of 46 equations with 46 unknowns: 22 values of $S_i$ and $Z_{al}$ each and the values of $N_T$ as well as $N_B$. The rolling parameters, results of calculation by the procedure given, by the proposed model and experimental data are given in Table 1. For the calculation of $P$, and $M_1$ values, the procedure was used.

The processing of data in Table 1 shows that when the procedure was used, the standard deviation

$$\Delta = \sqrt{\frac{\sum_{i=1}^{N-1} (S_{i}^C - S_{i}^A)^2}{N-1}}$$

of calculated values of the wall thickness $S_{i}^C$ from the actual values of this parameter $S_{i}^A$ was $\Delta = 0.150$ mm. When the present mathematical model was used, the value of $\Delta$ was 0.085 mm and for 1.76 times smaller. Hence, the rolling parameter calculation accuracy is improved when the proposed procedure is used.

7 CONCLUSION

The mathematical model of the continuous plugless tube rolling process has been developed and successfully tested. It improves the accuracy of calculation of the process parameters in comparison with the earlier developed procedure.

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8 REFERENCES


