This paper presents a numerical methodology aimed at identifying quantitatively the crazing under quasi-static impact loading of rigid PVC pipes. It also presents the correlation between the length of the crazed region and the area of the impacted region. Testing low-speed impacts has provided a damaging loading before the rupture of the sample’s pipe section. The striker impact on the pipes allows us to appreciate the damage caused. The craze in the impacted area was identified in our previous work\(^1\), the different criteria of the craze initiation are presented. The criterion of Steinstern and Myers, better adapted to rigid PVC\(^2\), is used in a numerical simulation on ANSYS for the problems of static contact. For tests carried out at low velocity, an equivalent static load representing the dynamic impact loading can account for the stress states in the structure of the pipe\(^3\). The analysis of the stress fields in the structure of the pipe compared to the stress of the craze initiation depends on the first stress invariant and allows us to deduce the length of the crazed region in the thickness of the pipe. Finally, we present the correlation between the length of the crazed region and the area of the zone of impact (the area of the imprint of the striker on the pipe).

**Keywords:** damage, crazing, impact, PVC, amorphous polymer
However, there are differences between them. Crazes are characterized by crack partitions linked together by small parallel fibrils of the material (see Figure 1a). Crazes and cracks proceed in three stages. Initiation, which is the enlargement and rupture of fibrils. Crazing involves a process of stretching the bulk of the material towards the area were the fibrils are located at the beginning of the crack length a.

Slika 1: (a) Diagram po{kovanja, ki ga karakterizirajo fibrili. (b) Zona risov z rezultati nastanka razpok. Zona A je podro~je, kjer je bil steklast polimer elasti~no deformiran. B je povr{ina nastanka risov. D je prelom fibrilov, ki rise spremeni v razpoko z dol`ino a.

Figure 1: (a) Diagram of a craze with cracks characterized by the presence of fibrils. (b) Craze region with the result of fissure formation. Zone A represents the region were the glassy polymer is elastically deformed. B, the area were initiation crazing occurs. C propagation or growth of the crazed region. D Rupture of the fibrils leading to the transformation of the craze into a crack of length a.

2.2 Criteria of craze initiation

In elastic and visco-elastic regimes or after the development of a significant plastic deformation, the corresponding formula of the initiation criteria is distinct for each regime. In a previous publication, we detected the first craze in elasto-visco-elastic regime for rigid PVC. The initiation criteria presented below, correspond to the craze appearances in elasto-visco-elastic regimes and elaborate on different phenomenological bases.

Following the observation of an incubation time for the craze formation where the constant stress is less than half of the yield stress ($\sigma_y$), Argon et al. proposed a sophisticated formula (1), as shown below:

$$|\sigma_1 - \sigma_2| \geq \frac{A}{C + 3I_1/2\sigma_y}Q$$

where A and C represent material parameters. $\sigma_1$ and $\sigma_2$, respectively, are the maximum and minimum principal stress. $I_1$ is the first stress invariant, $Q = 0.0133$ is a factor controlling the dependency of the criterion to the shear stress on $I_1$ and $\sigma_y$ is the yield stress.

Sternstein and Myers proposed a criterion based on the mechanism of micro-void formation in a dilatational stress field, and on the stabilization of micro voids through a deviatoric stress (2). Recently, following the same approach, Gearing proposed a criterion based only on the maximum principal stress (3):

$$\sigma_1 \geq A + \frac{B}{I_1}$$

$\sigma_1$ is the maximum principal stress.

With the same idea, Oxboough and Bowden suggested the definition of a criterion based on critical deformation. The criterion has been reformulated in the stress for an elastic material, with v standing for the Poisson’s ratio and E for the Young’s modulus ($X' = EX$ and $Y' = EY$ are two material parameters depending on the temperature) (4): $E

$$\sigma_1 - \sigma_2 \geq A + \frac{B}{I_1}$$

To account for the sensitivity to hydrostatic pressure observed for some polymers, Ishikawa proposed the following criterion (5):

$$I_1 \sigma_y = A + \frac{B}{I_1}$$

3 EXPERIMENTAL PROGRAM

3.1 Material

The pipes used are mostly made up of polyvinyl chloride (rigid PVC). They come from the same
molding. The pipe dimensions are 63 mm in diameter and a wall with a 4.5 mm thickness (see Figure 2).

3.2 Presentation of the impact machine

This device allows us to drop a weight (striker) from a selected height onto a pipe (see Figure 3a and 3b). The section of the pipe is placed on a support "V". A cylindrical rigid bar to help prevent the rebound of the striker during and after the impact is introduced. A weight of the striker equal to 16 kg and of radius $R = 50$ mm is used. (See Figure 3c). The height used ranges between 0 m and 2 m. The mass, height and velocity are used to calculate the kinetic energy of the impact.

After impact, the object dropped of mass 16 kg, leaves an elliptical mark called the impacted area or the area of impact at the point of impact (see Figure 4).

4 METHODOLOGY FOR DETERMINING THE LENGTH OF THE CRAZED REGION

In a previous publication 1 we have identified the crazing as the morphological mechanism of damage in the impacted area (see Figure 4). In this section, we propose a numerical methodology for determining the length of the crazed region in the pipe wall thickness below the impacted area (see Figure 4).

The craze initiating to the critical stress (in agreement with the criterion of craze initiation considered) and the methodology consist of delimiting areas in the pipe were the stress levels are greater than the critical stress ($\sigma \geq \sigma_{cr}^m$, $\sigma_{cr}^m$: critical stress initiation).

The different criteria for the craze initiation presented above (Section1.2) are valid, depending on the type of material. An analysis of these criteria showed that the criterion of Sternstein and Myers (2) is the best adapted for rigid PVC.

The determination the length of the crazed region requires the calculation of stress fields in the wall thickness of the pipe material at the time of impact. These experiments are difficult. We have constructed a numerical model based on Ansys for the simulation of the problem of impact dynamics 13. As our test impacts are at low velocities ($V < 6 \text{ m/s}$), it is shown in the literature that a static equivalence of a dynamic problem is sufficient to account for the stress in the structure 3. Therefore, the impact of the striker on the pipe is treated as a static contact problem between two solid bodies. The normal load applied on the upper surface of the striker (see Figure 5) is obtained through the equations of Hertz, linking the dimensions of the impacted area (see Figure 4) to the static normal force. A visco-elastic constitutive law is applied to the numerical model in order to reduce the computation times of the problem, which is already highly non-linear, without altering the objectives of the study. Indeed, the craze initiating for the case of rigid PVC in the elastic regime and the critical stress initiation are expected to lower the yield stress.

The high stress levels in the material (stress greater than the resistance of the material studied) do not affect the final results of the experiment.

Figure 3: (a) Impact machine from the German TestSystems UTS, (b) synoptic diagram of the impact machine, (c) dimensions of the hemispherical striker

Slika 3: (a) Udarna naprava nemškega proizvajalca TestSystems UTS, (b) sinoptričen diagram udarne naprave, (c) dimenzije hemisferičnega kladiva

Figure 4: Photographs of the impacted area on sections of the pipe at varying heights after the drop of the striker (mass)

Slika 4: Posnetka površine udara na prerezih cevi po različni višini padca udarnega kladiva

Figure 5: Geometric representation of the numerical model: (a) full actual geometry, (b) ¼ of the geometry implemented due to the symmetry of the problem.

Slika 5: Geometrična slika numeričnega modela: (a) polna realna geometrija, (b) ¼ geometrije, uporabljene zaradi simetrije problema
The tests impacts, being numerous, means that the details of the methodology for determining the length of the crazed region in the pipe wall thickness will be presented for an impact using a striker of mass 16 kg dropped from a height of 1.25 m (designated impact: 1.25 m/16 kg).

\( \sigma_{S&M} \), the stress parameter of the pipe to be found, corresponds to the criterion of Sternstein and Myers (2):

\[
\sigma_{S&M} = \left| \sigma_1 - \sigma_2 \right| \geq \sigma_{cr} = A^0 + \frac{B^0}{I_1} \tag{6}
\]

where \( \sigma_1 > \sigma_2 > \sigma_3 \), \( A^0 = 19.936 \) MPa and \( B^0 = 1 \) 203.384 MPa 2 for rigid PVC.

**Figure 7**, represents a cartograph stress state in the wall thickness of the pipe. Around each iso-value of a contour line with the stress represented by \( \sigma_{S&M} \) we observe that from the given equation above, the critical stress of Sternstein and Myers \( \sigma_{cr} \), see formula (6)) varies weakly with \( I_1 \) (see **Figure 6**). Moreover, it seems reasonable that the local critical stress for the craze initiation decreases as \( I_1 \) increases. Thus, the creation of micro-voids before crazing would be easier and would lead to a lower stress initiation. This reflects in a decrease in the critical stress while \( I_1 \) increases.

In **Figure 6** we see that the variation of is not very significant and in all cases it does not fluctuate above and below 10 % of its average value. Thus, the critical stress \( \sigma_{cr} \) at all points on an iso-value contour of the stress \( \sigma_{S&M} \) is considered constant.

Each iso-value contour of the stress in **Figure 7b** is a possible limit between the crazed regions and others not crazed in the thickness of the pipe. This limit is identified when. We also notice that all the iso-value contours cover the entire section of the thickness of the pipe (see **Figure 7b**). Thus, whatever the limit of the iso-value contour, crazing occupies the entire thickness of the pipe, where the stress fields represent (with being constant on an iso-value contour).

**Figure 7**: Cartograph of stress states \( \sigma_{S&M} = |\sigma_1 - \sigma_2| \) corresponding to the criterion of Sternstein and Myers. (a) \( 1/4 \) of the geometry of the pipe, (b) surface \( S_1 \) of the thickness of the pipe

**Slika 7**: Kartografija napetostnega stanja \( \sigma_{S&M} = |\sigma_1 - \sigma_2| \)

Considering as a constant on an iso-value contour, the representation of the stress states compared to critical stress initiation along an imaginary line (see **Figure 7b**) is sufficient to determine the length of the crazed region, which corresponds to .

**Figure 8**, represents the stress states compared to along an imaginary line located at any position, crossing the right section of the structure of the pipe (\( S_1 \)) just below the contact point (see **Figure 7**).

In **Figure 8**, the intersection between the plot and indicates the limit of the crazed region. Beyond this intersection, the stress is less than the stress initiation. The stress state is abnormally large and the structure can be attributed to the elastic constitutive law applied to the numerical model. Nevertheless, the intersection point is...
different impact tests performed (lengths of the crazed regions corresponding to the impacted area (see Figure 4a). However, it does not help to determine its length. But the developments of these two parameters show correlations (see Figure 9).

In the range of the impact tests performed \((h \in [0.2 \text{ m}, 1.7 \text{ m}])\), the evolutions of \(a_c\) and \(a\) show an almost linear growth with the level of impact (see Figure 9b). The area of the imprint (impacted area) may be a good indicator of the length of the crazed region in a pipe subjected to an impact.

5 CONCLUSION

A numerical methodology for determining the length of the crazed region in a rigid PVC pipe subjected to a quasi-static impact has been presented.

With crazing identified on the impacted area \(^1\), determining its length with the methodology proposed involves a comparison of the stress states in the structure for the critical stress of the craze initiation. After choosing the criterion of initiation of Sternstein and Myers \((6)\) for rigid PVC \(^2\) we notice that its dependence on the first stress invariant \(I_1\) induces some distinct critical values. However, on an isovalue contour of stress, the criterion does not deviate more than \(\pm 10\%\) from the average value (see Figure 6). From this small variation we considered the value of the critical initiation as a constant on an isovalue contour. Thus, the comparative representation of the stress in the wall thickness of the pipe \(\sigma_{S&M}\) and the critical stress initiation craze \(\sigma_{cr}^m\) (see Figure 8) along a line crossing all the boundaries of all the isovalue contours (see Figure 7b) allow us to deduce the length of the crazed region corresponding to \(\sigma_{S&M} \geq \sigma_{cr}^m\). Depending on the level of the impact load, we see a linear growth of the length of the crazed region (see Figure 9a), which is linearly correlated to the size of the corresponding impacted area (see Figure 9b).

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