NUMERICAL SOLUTION OF HOT SHAPE ROLLING OF STEEL

NUMERIČNA REŠITEV VROČEGA VALJANJA JEKLA

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1 INTRODUCTION

The main aim of this paper is elaboration of the coupled thermo-mechanical computational model developed for hot shape rolling of steel. The output of the thermal model is the temperature field and mechanical model the displacement (deformation). Shape rolling is a 3D process, however it is analyzed with 2D imaginary slices which is denoted as a slice model. The coordinate system of a 2D slice is based on Langrangian description where the slice travels across the rolling contact. The third axis, the rolling direction, is based on the Eulerian description where there is a constant inflow and outflow of steel through the rolling direction. This is considered as a mixed Eulerian-Lagrangian model. It was discussed previously by many authors. The meshless method, based on collocation with radial basis functions is used to solve the thermo-mechanical problem. The node distribution is calculated by an elliptic node generation at each deformation step to the new form of the billet. The solution is calculated in terms of temperatures and displacements at each node. Preliminary numerical examples for the new rolling mill in Store Steel are shown.

Keywords: steel, hot rolling, radial basis functions, meshless numerical method


Ključne besede: jeklo, vroče valjanje, radialne bazne funkcije, brezmrežna numerična metoda

2 THERMAL MODEL

The thermal model of the shape rolling process is aimed to calculate the temperature field of the steel slab during the rolling process. The three dimensional domain \( \Omega^{3D} \) with boundary \( \Gamma^{3D} \) is considered. The solution procedure is based on Cartesian coordinate system with axes \( x, y, z \). Slices coincide with coordinates and the rolling direction is \( z \). The steady state temperature distribution in the rolled product is defined through the following equation.

\[
\nabla \cdot (\rho c_p \nabla T) + S ; p \in \Omega^{3D} (x, y, z) \quad (1)
\]

Since we analyze the process with 2D slices perpendicular to the rolling direction and assume that uniform velocity over the slices (homogenous compression) takes place. The Equation (1) can be transferred into

\[
\rho c_p \frac{dT}{dt} = \nabla \cdot (k \nabla T) + S ; p \in \Omega^{2D} (x, y) \quad (2)
\]

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with \( p, \rho, t, \epsilon_p, T, k, \nu_{\text{entry}}, A_{\text{entry}}, (A), \) and \( S \) standing for position vector, density, time, specific heat, temperature, thermal conductivity, entry speed of billet, entry cross sectional area, instant cross sectional area and internal heat generation due to plastic deformation. It is assumed in the slice model that the heat transport takes place only in the direction perpendicular to the rolling direction and that the homogenous deformation takes place. The Neumann boundary condition on the part of the boundary denoted as \( \Gamma^N \). Robin boundary condition on the part of the boundary denoted as \( \Gamma^R \) are taken into consideration \( (\Gamma = \Gamma^N \cup \Gamma^R) \) which are described below,

\[
-k\nabla T(p) \cdot n_r = -k \frac{\partial T(p)}{\partial n_r} = q ; \quad p \in \Gamma^N \quad (3)
\]

\[
-k\nabla T(p) \cdot n_r = -k \frac{\partial T(p)}{\partial n_r} = h[T(p) - T_0(p)] ; \quad p \in \Gamma^R \quad (4)
\]

The \( N_{\Omega} \) nodes at the domain and \( N_{\Gamma} \) nodes at the boundary are used to discretize the temperature in LRBFCM where for each node \( p_n = (p_x, p_y)^T \). For each node there is a defined influence domain with \( N_{\omega} \) neighboring nodes. For each influence domain a radial basis function in terms of multiquadric is written

\[
y_i = \sqrt{(p_x - p_{x,n})^2 + (p_y - p_{y,n})^2 + c^2}/a_{max} + c^2
\]

The temperature can now be interpolated as

\[
T = \sum_{n=1}^{N_{\Omega}} \psi_n(p) \alpha_n
\]

with the collocation coefficients to be determined. The main equation in 2D can be rewritten by using the explicit time stepping,

\[
\rho c_p \frac{T_{n+1} - T_n}{\Delta t} = \nabla k \cdot \sum_{n=1}^{N_{\Omega}} (\nabla \psi_n(p) \alpha_n) + \frac{\nabla k}{\Delta t} \sum_{n=1}^{N_{\Omega}} (\nabla \psi_n(p) \alpha_n) + S
\]

\[
(5)
\]

3 MECHANICAL MODEL

A strong form is chosen here for analysis due to its compatibility with LRBFCM. A domain \( \Omega \) with boundary \( \Gamma = \Gamma^N \cup \Gamma^R \) is considered where \( \Gamma^R \) is the essential and \( \Gamma^N \) the natural boundary condition. The strong formulation of the static metal deformation problems is:

\[
L \sigma + b = 0
\]

(6)

In the calculations, in order to avoid complications of a 3D solution, the slab is analyzed, compatible with the thermal model, with imaginary traveling 2D slices that are perpendicular to the rolling direction. For a 2D slice method, \( L \) is the 3x2 derivative matrix with elements \( L_{11} = \partial / \partial p_x, L_{12} = 0, L_{21} = \partial / \partial p_y, L_{22} = \partial / \partial p_y, \) and \( L_{31} = \partial / \partial p_x \), \( \sigma = [\sigma_x, \sigma_y, \sigma_z] \) is the stress vector, and \( b = [b_x, b_y, b_z]^T \) is the body force. At the essential boundary \( \Gamma^E \)

\[
u(p) = \bar{u}(p) ; \quad p \in \Gamma^E \quad (7)
\]

where \( \bar{u}(p) \) is the prescribed displacement vector and \( \bar{u}(p) \) is the prescribed displacement vector. At the natural boundary condition \( \Gamma^N \)

\[
N^T \sigma = \bar{T} ; \quad p \in \Gamma^N \quad (8)
\]

is valid, where \( \bar{T} \) is the prescribed surface traction \( \bar{T}=[T_x, T_y]^T \). \( N \) is the 2x3 matrix of direction cosines of the normal direction at the boundary which can be defined as \( N_{11} = N_{12} = n_x, N_{13} = N_{21} = 0, N_{31} = N_{22} = n_y \) \( (n_x, n_y) \) represent correlation of the normal at the boundary). In a 2D system the equations for mechanical model can be written as,

\[
\frac{\partial \sigma_x}{\partial p_x} + \frac{\partial \sigma_y}{\partial p_y} + b_x = 0, \frac{\partial \sigma_y}{\partial p_x} + \frac{\partial \sigma_x}{\partial p_y} + b_y = 0
\]

(9, 10)

The discretization is made in terms of displacement on \( x \) and \( y \) axes for each slice,

\[
u_x(p) = \sum_{n=1}^{N_{\Omega}} \psi_n(p) \alpha_n, \quad \nu_y(p) = \sum_{n=1}^{N_{\Omega}} \psi_n(p) \alpha_n
\]

(11, 12)

Since the strain vector \( e = [\varepsilon_x, \varepsilon_y, \varepsilon_z] \) can be written in terms of displacement as \( \varepsilon = Lu \), the strain vector \( \sigma \) can be expressed as a stress vector by using 6x6 stiffness matrix \( C \) which depends on the material characteristic assumption such as elastic, elastic-plastic or ideally plastic.

\[
\sum_{n=1}^{N_{\Omega}} C_{ij} \frac{\partial \psi_n}{\partial p_i} + C_{ij} \frac{\partial \psi_n}{\partial p_j} \frac{\partial \psi_n}{\partial p_j} + (C_{ij} + C_{ij}) \frac{\partial \psi_n}{\partial p_i} \frac{\partial \psi_n}{\partial p_j} + b = 0
\]

(13)
4 TRANSFINITE INTERPOLATION (TFI)

This technique is used to generate initial grid which is confirming to the geometry encountered in different stages of plate and shape rolling. Suppose that there exists a transformation \( \mathbf{r}(p_i, p_0) = \{pA(p_i, p_0), pB(p_i, p_0)\}^T \) which maps the unit square, \( 0 < p_i < 1, 0 < p_0 < 1 \) in the computational domain onto the interior of the region ABCD in the physical domain such that the edges \( p_0 = 0.1 \) map to the boundaries AB, CD and the edges \( p_0 = 0.1 \) are mapped to the boundaries AC, BD. The transformation is used for this purpose is defined as

\[
\mathbf{r}(p_i, p_0) = \begin{pmatrix}
(1 - p_0)\mathbf{r}(p_i) + \mathbf{r}_t(p_0)\mathbf{r}_s(p_i) + \\
(p_i)(1 - p_0)\mathbf{r}_t(0) - (1 - p_i)p_0\mathbf{r}_t(0) - \\
(1 - p_0)p_0\mathbf{r}_t(1) - p_0\mathbf{r}_t(1)
\end{pmatrix}
\]

Where \( \mathbf{r}_t, \mathbf{r}_s, \mathbf{r}_r, \mathbf{r}_l \) represent the values at the bottom, top, left and right edges respectively. The initial grid is refined through ENG. Figure 2 shows initial node generation through TFI and its correlation with ENG.

5 CONCLUSION

In this paper the thermal and mechanical formulations are given for hot shape rolling. The numerical method for the solution of the problem is based on meshfree LRBFCM. The preliminary result of mechanical model for elastic case is presented in Figure 3. The future work will include plastic deformation in a sequence of 10 rolling stands as recently installed in Store–Steel Company.

6 REFERENCES