CALCULATION OF THE LUBRICANT LAYER FOR A COARSE SURFACE OF A BAND AND ROLLS

IZRAČUN SLOJA MAZIVA NA GROBI POVRŠINI TRAKU IN VALJEV

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The effect of the average roughness of a lubricated band caused by dressing processes is analysed by applying the Reynolds differential equation for lubrication with the incorporated average roughness and evolution in the Fourier series to the third member. The analysis has shown that the average roughness has two effects on the lubricant-layer thickness in the entering section of the deformation zone. For a small surface roughness, the nominal lubricant-layer thickness decreases slowly (if the process is treated as occurring on a smooth surface) and the thickness grows again with an increase in the roughness. The basis for the analysis was the numerical Monte-Carlo method and the developed approximate analytical solution was in acceptable agreement with the numerical method.

Keywords: surface roughness, lubricant-layer thickness, Reynolds equation, Monte-Carlo method, Fourier series

1 INTRODUCTION

This technology is strongly associated with the quality of technological lubricants as it:
- diminishes the contact friction,
- removes the heat, cools the tool and diminishes the wear,
- diminishes the deformation resistance and the deformation work,
- diminishes the sticking to the tool and keeps the surface of the product clean.

The basic groups examined in this work1–3 are:
- liquid emulsions,
- fats and compounds,
- consistent lubricants,
- transparent/glass lubricants,
- powder lubricants and
- metallic lubricants.

Technological lubricants must meet a series of requirements, beginning with a high lubricity – the ability to form a flat, firm layer separating the contact surfaces – then there are thermal consistency and stability that prevent the damaging effect of the product corrosion, the properties not posing any health and environmental risks, etc.

The liquid emulsions, whose compounds are mixtures of vegetable and mineral oils, are especially used in the cold rolling of 0.3–0.4 mm thick sheets and strips.

In the cold rolling of sheets and strips, the dressing process is also used with an application of liquid lubricants to reduce undulation.

2 MATHEMATICAL MODELLING

Mathematical modelling is a requirement of today’s metallurgy4,5 and it is also used in the field of plastic deformation of metals. For an analysis of smooth surfaces6,7 the following equation is used:

\[
\frac{dp}{dx} = 6\mu(\nu_0 + \nu_r) \frac{12\mu Q}{\varepsilon^2(x)}
\]  
(1)

\[ Q(x) = \int_0^{\varepsilon(x)} ud\gamma = \frac{1}{12\mu} \frac{dp}{dx} \varepsilon^3(x) + \frac{(\nu_0 + \nu_r)}{2} \varepsilon(x) \]  
(2)

The geometry of the lubricant contact6 and the length of the lubricant wedge are described with the relations (3), (4) and (5):

\[
\varepsilon(x) = \varepsilon_0 + R \left[ \cos \alpha - \sqrt{1 - \left( \sin \alpha \frac{x}{R} \right)^2} \right]
\]  
(3)
Reflexion of sheet roughness is added, as \( \varepsilon_0 \), to the lubricant wedge (4). The calculation is possible only with numerical mathematical methods and, in the program MATHEMATICA, the numerical method Monte Carlo was used. In the theoretical calculations regarding the model of the average roughness, the following function developed to the third term of Fourier series was applied:

\[
\delta(x) = \frac{4}{\pi} \left( \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x \right) R_z
\]

(8)

3 RESULTS AND DISCUSSION

In Table 1 the standard values of geometrical, rheological and kinematic characteristics of the processes of theoretical investigations are given according to the Russian-Ukrainian\(^{10,11}\) authors.

### Table 1: Standard lubricant characteristics for theoretical calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )-piezo coefficient of viscosity</td>
<td>2.18E-7</td>
<td>Pa(^{-1} )</td>
</tr>
<tr>
<td>( \rho_{0_{w}} )-rolling pressure</td>
<td>20E6</td>
<td>Pa</td>
</tr>
<tr>
<td>( \nu_{R} )-circumferential roll speed</td>
<td>10</td>
<td>m/s</td>
</tr>
<tr>
<td>( \nu_{0_{s}} )-sheet speed</td>
<td>6</td>
<td>m/s</td>
</tr>
<tr>
<td>( R )-roll radius</td>
<td>0.35 (0.25)</td>
<td>m</td>
</tr>
<tr>
<td>( \alpha )-gripping angle</td>
<td>0–0.02</td>
<td>rad</td>
</tr>
<tr>
<td>( \varepsilon_{0_{z}} )-lubricant thickness on sheet</td>
<td>0.001–0.00001</td>
<td>m</td>
</tr>
<tr>
<td>( A )-technological parameter</td>
<td>1965512 (3934525)</td>
<td>m(^{-1} )</td>
</tr>
<tr>
<td>( R_z )</td>
<td>1–10</td>
<td>( \mu ) = 1–10 ( \mu ) mm</td>
</tr>
</tbody>
</table>

The parameters in Table 1 are of two groups:
1- lubricant rheological characteristics (\( \mu_0, \gamma \))
2- geometrical characteristics of the technological process (\( R, \alpha, R_z \))
3- kinematics (\( \nu_0, \nu_R \))
The solutions of differential equation (6) are partially given in Table 2.

The examined roughness is classified in 10 vertical classes and the band profile roughness in 32 horizontal classes.

In principle, with a decreasing band-lubricant thickness, the lubricant thickness in the entering section of the metal deformation zone is also decreased (\( \varepsilon_0 \)). As shown in Figure 1, the lubricant wedge has the ideal geometry and can give economic savings of the lubricant in the metalworking technology.

The numerical integration of equation (6) was checked with the approximate analytical solutions possible in the case of practical interest, which is found in equations (9), (10a)–(10e) and (11). Equation (9) is the simplest analytical solution that does not consider the thickness of the band lubricant layer, \( \varepsilon_0 >> \varepsilon_0 \). With a clear complexity, equation (11) corrects this deficiency:

\[
315AR^2\alpha^2 - 168R^2\alpha^4 - 1824\delta^2 = 0
\]

\[
\varepsilon_0 = 0.5Ra^2 \ldots A = \frac{1 - \exp(-\gamma p_0)}{6\nu(v_0 + v_R)}
\]

\[W_i = A = \left[ \frac{4}{3\pi R} - \frac{R}{3} \right]
\]

\[-W_j = 3b\]

\[
W_3 = \frac{8}{5\alpha R} - \frac{R}{8}
\]

\[-W_4 = 3R
\]

In Table 3 approximate numerical and analytical solutions are compared. The approximate numerical solutions can be compared with numerical integration only for the entering roughness profile, thus, at the entering section of the deformation zone with \( x = 0 \).

It is clear from Table 3 that the simple analytical form of equation (9) with numerous approximations describes well the lubricant layer for the case of a lubricant excess on the sheet and the rolls.

The longitudinal band profile on abscissa is shown in 66 classes and on ordinate in 11 classes for roughness (0–10 μm). It is useful to calculate the lubricant thickness \( \varepsilon_0 \) in the range of 8.5–12.5 μm in the area of I–I. Q, K and W designations connect the specific areas of the network diagram with the contour plot (an aircraft picture of the network diagram).

B and C are the left and right sides of the band roughness defined as a sine evolution function in the range of (π–2π) rad and C in (0–7π) rad.

Line P in Figure 2 represents the nominal lubricant-layer thickness on side C, thus, by having the thickness for \( R = 6 \mu m \), an equivalent to the lubricant-layer thickness on a smooth surface is obtained. Side B does not have this property.
In Figure 3 both sides of the roll longitudinal roughness C from Figure 2 are shown. The average roughness conserves the same properties as in Figure 2. The longitudinal roughness profile in the range of classes 33 to 66 gives a more stable hydrodynamic lubrication, while for classes 1 to 33 the hydrodynamic lubrication is already seriously impaired by the low roughness of the band and rolls. The lubricant layer decreases rapidly and spreads to fractal areas. A stable lubrication can be achieved on small band segments and around class 4 of the longitudinal sheet profile and around classes 10 and 30. The complex shapes of the lubrication space are probably determined by the band and roll roughness in the entering section of the deformation zone that determines a different lubrication layer than in the case of smooth-sheet and roll surfaces.

### 4 CONCLUSIONS

Based on the results of theoretical analyses of the effect of the band roughness on the lubrication dressing processes, the following conclusions are proposed:

- The average band roughness has a critical value when it starts to affect positively the lubricant layer with its increase in comparison with a smooth surface. Up to line P in Figure 2, the lubricant layer has a tendency to increase and to decrease the formation of sunk baskets in area Q. The theoretical explanation for this is that the surface roughness determines the shape of the lubricant layer for every value of $R_z$. This is the range of a stable lubrication.

- If congruous roll roughness is added to the average band roughness, forming a longitudinal roll roughness with the positive side in the range of $(0-\tau)$, the thickness of the lubricant layer in the entering section of the band deformation zone will increase its longitudinal profile from class 33 to 66 (Figure 3 and Table 4) and will approach the boundary lubrication.

- The developed approximate analytical solutions agree with the numerical integration of equation (6) and ensure a reliable approach to the analysis.

- If the technological process was performed with a nominal lubricant-layer thickness marked with line P in Figure 2 the best roll rhythm would be obtained without significant fluctuations of the lubricant thickness, especially in the case of the boundary-lubrication proximity. This includes the control of the roll roughness.

### 5 SYMBOLS AND FIGURES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_0$</td>
<td>m, (μm)</td>
<td>Lubricant thickness in the entering section of the deformation zone (Figure 1)</td>
</tr>
<tr>
<td>$\varepsilon(x)$</td>
<td>m</td>
<td>Lubricant thickness in the range of $[-a : 0]$, Figure 1, equations (3) and (5)</td>
</tr>
<tr>
<td>$\varepsilon_a$</td>
<td>m</td>
<td>Lubricant thickness ahead of the entering section of the deformation zone</td>
</tr>
<tr>
<td>$a$</td>
<td>m</td>
<td>Length of the lubricant wedge (Figure 1), equation (4)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>rad</td>
<td>Band dressing angle</td>
</tr>
<tr>
<td>$v_R$</td>
<td>m/s</td>
<td>Circumferential roll speed</td>
</tr>
</tbody>
</table>
\begin{tabular}{|l|l|}
\hline
\textbf{\(v_T\)} & m/s & Mandrel speed \\
\hline
\textbf{\(R\)} & m & Roll radius \\
\hline
\textbf{\(R_s\)} & m & Roughness of the band surface, equation (8) \\
\hline
\textbf{\(\delta^2\)} & & Dispersion roughness of the sheet and rolls according to equation (9) \\
\hline
\textbf{\(\delta_s\)} & & Casual lubricant thickness depending on the band roughness (and rolls) \\
\hline
\textbf{\(< >\)} & & Operative mathematical expectation \\
\hline
\textbf{\(x, y\)} & & Descartes coordinates \\
\hline
\textbf{\(Q(x)\)} & & Volume use of lubricant (on the band perimeter) \\
\hline
\textbf{\(\mu_0\)} & Pa & Lubricant dynamic viscosity by the rolling pressure \\
\hline
\textbf{\(\mu\)} & Pa & Lubricant dynamic viscosity by the air pressure \\
\hline
\textbf{\(u\)} & m/s & Lubricant rate on the abscissa \\
\hline
\textbf{\(\gamma\)} & m\(^2\)/N & Piezo coefficient of lubricant viscosity \\
\hline
\textbf{\(p\)} & Pa & Rolling pressure \\
\hline
\textbf{\(Q\)} & m/s & Use of lubricant on the mandrel perimeter – a one-dimensional model \\
\hline
\textbf{\(dp/dx\)} & Pa/m & Pressure gradient in the lubricant layer, equation (1) \\
\hline
\textbf{\(\sin \alpha\)} & rad & Marking the trigonometric function for the gripping alpha angle \\
\hline
\textbf{\(H\)} & m & Enter band thickness \\
\hline
\textbf{\(h\)} & m & Exit band thickness \\
\hline
\textbf{\(A\)} & m\(^{-1}\) & Technological parameter: \(A = [1 - \exp(-\gamma p)] / \left[ 6\mu_0(v_s + v_t) \right]\) \\
\hline
\textbf{\(\exp, \pi\)} & & Base of natural logarithm (3.141) \\
\hline
\textbf{\(14\)} & & Reference \\
\hline
\textbf{\(1 \mu m\)} & & Micrometre \\
\hline
\textbf{\(S\)} & \(\mu m\) & Band- and roll-roughness classes \\
\hline
\textbf{\(L\)} & \(\mu m\) & Longitudinal holding-band profile \\
\hline
\textbf{\(Q, K, W\)} & & Markers for Figure 2 \\
\hline
\end{tabular}

6 REFERENCES


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