1 INTRODUCTION

Mathematical modelling is an essential and economical tool in the application of new techniques for the hot plastic deformation of metals and alloys. It makes it possible not only to reduce the number of technical experiments, but also to assess the interaction of the technological parameters of the investigated processes determining the usability of the final products. Therefore, it integrates, in most cases, the thermal, mechanical and microstructural processes. In the future modelling will constitute the basis of a system for controlling most technological processes. The constructed models will be applied simultaneously in an off-line simulation and on-line control of the processes.

The determination of the parameters of force in thermomechanically controlled processing (TMCP), particularly in the controlled rolling of new profiles, requires a knowledge of the values of the flow stress of the steel under the specific conditions of the deformation. One of numerous methods for determining these stresses is the hot plastometric torsion test, which also permits a physical simulation of the sequential rolling deformation. Thus, it becomes possible to determine the values of the flow stress in successive roll passes, separated by interpass times in which thermally activated processes occur, removing the results of the strain hardening.

The paper presents possibilities for the mathematical modelling of hot plastic deformation concerning some selected products obtained by rolling structural steels with micro-additives in the range of technological parameters simulating such processes in the plastometric torsion test. Special attention was paid to the verification of the rheological model developed by C. M. Sellars et al. describing the flow stress as a function of the deformation and temperature, as well as strain rate and the effect of dynamic recrystallization, which dominates in the hot deformation for the investigated structural steels with micro-additives of Nb, V, Ti and N.
2 EXPERIMENTAL PROCEDURE

2.1 Materials

The tests were carried out on industrial melts of structural steels with micro-additives, the chemical composition of which is shown in Table 1.

Plastometric tests of the steel were performed applying the hot-torsion method. The tested samples had a diameter of 6 mm and a basis length of 10 mm. They were cut out from slabs with dimensions of 200 mm × 220 mm × 5300 mm provided after the hot rolling. By means of continuous torsion, the characteristics of plasticity for the investigated steels were determined in means of continuous torsion, the characteristics of plastometric tests of the steel were performed applying the hot-torsion method. The tested samples had a diameter of 6 mm and a basis length of 10 mm. They were cut out from slabs with dimensions of 200 mm × 220 mm × 5300 mm provided after the hot rolling. By means of continuous torsion, the characteristics of plasticity for the investigated steels were determined in means of continuous torsion, the characteristics of plastometric tests of the steel were performed applying the hot-torsion method. The tested samples had a diameter of 6 mm and a basis length of 10 mm. They were cut out from slabs with dimensions of 200 mm × 220 mm × 5300 mm provided after the hot rolling.

2.2 Rheological model

In constitutive equations, actually applied for the purpose of modelling the processes of hot plastic working, the effects of dynamic recovery and dynamic recrystallization are not always conveyed explicitly. The equation in which both components are distinctly separated, elaborated at the University of Sheffield by C. M. Sellars et al.,5,6, takes the following form:

\[ \sigma = \sigma_0 + \left( \sigma_{SS(e)} - \sigma_0 \right) \left( \frac{\varepsilon}{\varepsilon_0} \right)^{1/2} - R \]  \hspace{1cm} (1)

where the respective variables are defined as follows:

\[ R = \begin{cases} 0 & \text{for } \varepsilon \leq \varepsilon_C \\ \left( \sigma_{SS(e)} - \sigma_0 \right) \left( 1 - \exp \left( - \left( \frac{\varepsilon}{\varepsilon_0} \right)^2 \right) \right) & \text{for } \varepsilon > \varepsilon_C \end{cases} \]  \hspace{1cm} (1a)

\[ \sigma_0 = \frac{1}{\alpha_0} \sinh^{-1} \left( \frac{Z}{A_0} \right)^{1/2} \]  \hspace{1cm} (1b)

The variables in Equation (1) are as follows:

\[ \sigma_0 \] – flow stress
\[ \sigma_{SS(e)} \] – the maximum stress when the plastic strain \( \varepsilon = 0 \)
\[ \sigma_{SS(e)} \] – the onset of steady-state conditions in the extrapolated curve
\[ \sigma_{SS(e)} \] – the onset of steady-state conditions on the experimental flow stress curve
\[ \varepsilon \] – plastic deformation
\[ \varepsilon_C \] – strain for the onset of dynamic recrystallization
\[ \varepsilon_C \] – the "transient strain constant" and effectively defines the curvature of the flow-stress curve between \( \sigma_0 \) and \( \sigma_{SS(e)} \) where the equation saturates
\[ \varepsilon_R \] – the strain required to reach a fixed amount of softening, measured in terms of \( \Delta \sigma / \Delta \sigma_0 \). This term effectively defines the rate of softening as a result of the dynamic recrystallization
\[ \varepsilon_{SS(e)} \] – the strain at the "onset" of steady state when dynamic recrystallization occurs
\[ \varepsilon \] – strain rate
\[ Z \] – Zener – Hollomon parameter
\[ R \] – the universal gas constant
\[ R \] – a term expressing the dynamic recrystallization
\[ Q_{def} \] – the activation energy for the deformation
\[ A, \alpha, \eta, q, C, N \] – constants for each characteristic stress \( \sigma_p \).

Table 1: Chemical composition of the tested steels

<table>
<thead>
<tr>
<th>Steel</th>
<th>C</th>
<th>Mn</th>
<th>Si</th>
<th>Cr</th>
<th>P</th>
<th>S</th>
<th>Nb</th>
<th>V</th>
<th>Ti</th>
<th>N</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.15</td>
<td>1.03</td>
<td>0.25</td>
<td>–</td>
<td>0.018</td>
<td>0.009</td>
<td>0.017</td>
<td>0.05</td>
<td>–</td>
<td>0.0070</td>
<td>0.040</td>
</tr>
<tr>
<td>B2</td>
<td>0.16</td>
<td>1.25</td>
<td>0.32</td>
<td>–</td>
<td>0.023</td>
<td>0.019</td>
<td>0.030</td>
<td>0.01</td>
<td>–</td>
<td>0.0060</td>
<td>0.042</td>
</tr>
<tr>
<td>S9</td>
<td>0.38</td>
<td>1.52</td>
<td>0.63</td>
<td>0.56</td>
<td>0.027</td>
<td>0.012</td>
<td>0.030</td>
<td>–</td>
<td>0.11</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>S1</td>
<td>0.66</td>
<td>1.02</td>
<td>0.26</td>
<td>0.30</td>
<td>0.029</td>
<td>0.021</td>
<td>–</td>
<td>0.10</td>
<td>–</td>
<td>0.0080</td>
<td>–</td>
</tr>
</tbody>
</table>
The presented model illustrates more distinctly the behaviour of the material in the course of dynamic recrystallization, because it makes it possible to describe the point of deflection \((\varepsilon_{\text{peak}})\) on the curves \(\sigma - \varepsilon\) and the procedure of flow stress beginning at the peak strain up to the achievement of the value of the stresses in the steady state \((\sigma_{\text{ss}})\). The coefficients of this constitutive equation are usually determined based on the results of plastometric tests within a wide range of temperature and strain rates.

This allows us to model various structural materials over a wide range of conditions for hot plastic deformation. The main difficulty in applying this model lies in the large number of parameters, which have to be unambiguously identified. One of the main factors limiting the application of the mode simulating the processes of plastic working is the difficulty in determining these coefficients, the values of which depend on the kind of applied material. Nevertheless, Equation (1) has been used in this study for the investigated microalloyed steels.

For the optimization of the parameters of the applied model of C. M. Sellars\(^5,6\), describing stress–strain curves, making use of the goal function \(\phi\) in the form:

\[
\phi = \sum_{i=1}^{N_p} \frac{1}{N_p} \left( \frac{\sigma_{\text{m}} - \sigma_{\text{c}}}{\sigma_{\text{m}}} \right)^2
\]  

(2)

where:

- \(\sigma_{\text{m}}\) – measured stress
- \(\sigma_{\text{c}}\) – calculated stress
- \(N_p\) – number of measurement points of all the curves in the \(i\)-th experiment

For the purpose of the minimum of Equation (2), the Nelder and Mead Simplex algorithm was used. The calculations were performed by means of Scilab’s calculation packet.\(^9\) In results of the optimization for the appropriate parameters of Equation (1) were found.

3 EXPERIMENTAL RESULTS AND DISCUSSION

The results of the mathematical modelling of the process of high-temperature plastic deformation for the investigated micro-alloyed steels allowed us to verify analytically the assumed Equation (1) of the type \(\sigma_p = \sigma_\Phi(\varepsilon, \dot{\varepsilon}, T)\) describing the flow stress \((\sigma_p)\) on the experimental flow–stress curves, making use of the plastometric method for metals and alloys with a low stacking-fault energy (SFE), which in the course of hot deformation display the phenomenon of dynamic recrystallization. Modelling was applied for some selected microalloyed steels of the type HSLA with various contents of the micro-additive Nb (steel B1 and B2), as well as an average-carbon structural steel (0.38 % C) with a binary system of micro-additives of Nb and Ti and a structural rail steel containing about 0.66 % carbon with the micro-additive V (0.10 %). The performed numerical calculations were based on the results of a hot-torsion test, analysing about 80 stress-strain curves recorded with a wide range of external variables, particularly the temperature and the strain rate. A comparison of the experimental flow curves with those in the model concerning the investigated steels is presented in the diagrams in Figures 1 to 4. In order to obtain a universal description of the flow curves, the complete data of measurements \(\sigma_p(\varepsilon, \dot{\varepsilon}, T)\) concerning the respective kinds of steel were optimized. The influence of the varying distribution of strains on the radius of the twisted sample, as well as the changes in the temperature at the cross-section of a massive sample have, in the process of optimization, not been taken into account. This might perturb the obtained results, particularly the identified parameters of the rheological model, as has been shown in\(^10\). The elimination of the effect of the homogeneity of deformation and other errors in the measurements ensures the identification of the parameters of the model, applying the inverse method\(^11,12\). The paper\(^16\) analyses, however, only the application of this method in the case of various types of hot-compression tests. Future plans will include the application of the inverse method in dealing with the results of hot torsion tests. The numerically determined values of the coefficients assumed in the model, taking into account the Simplex algorithm.
concerning the analysed stress-strain curves are shown in Table 2, which also contains the values of the energy of activation of the process of deformation and the final values of the goal function $\phi$, representing the accuracy of the modelling solution. Due to measurement errors and limitations with the model, different values were obtained for the goal function (Equation (2)) concerning the particular types of investigated steels. However, the median square error of the calculation remains within 2–5 % limits. This proves that the assumed model fits well with the experimental results. The flow curves of the investigated steels indicate principally the characteristic procedure of the continuous dynamic recrystallization. Their analysis permits us to state that this process is essentially influenced by the temperatures (Figure 1) and the strain rate (Figure 2), as well as the chemical composition of the steel (Figures 1 to 4). The effect of these factors concerns, first of all, changes in the value of the maximum flow stress and the stresses in the steady state ($\sigma_{ss}$), and also the values of critical deformations ($\varepsilon_f$), generally assumed to be in compliance with the criterion of the plasticity of steel in the process of hot deformation. It has been generally assumed that the stresses in the flow of the investigated steels increase with the drop in the temperature and the increasing strain rate. It has also been found that the best compatibility in matching the experimental and theoretical curves is achieved in the case of steel S9 (Figure 3), with the

![Figure 2: Experimental and modelled flow curves of microalloyed steel B2, hot twisted at $T_{\text{def}} = 900 \, ^\circ C$ and a strain rate of: a) $\dot{\varepsilon} = 0.8 \, s^{-1}$ and b) $\dot{\varepsilon} = 4.45 \, s^{-1}$](image2)

Slika 2: Eksperimentalna in modelirana krivulja teženja jekla B2, vroče zvijanega pri $T_{\text{def}} = 900 \, ^\circ C$ in hitrosti deformacije: a) $\dot{\varepsilon} = 0.8 \, s^{-1}$ in b) $\dot{\varepsilon} = 4.45 \, s^{-1}$

![Figure 3: Comparison of experimental and modelled flow curves of steel S9 in a hot torsion test: a) $T_{\text{def}} = 900 \, ^\circ C$, $\dot{\varepsilon} = 1 \, s^{-1}$, b) $T_{\text{def}} = 1000 \, ^\circ C$, $\dot{\varepsilon} = 10 \, s^{-1}$](image3)

Slika 3: Primerjava eksperimentalne in modelirane krivulje teženja jekla S9 pri preizkusu vroči torzije: a) $T_{\text{def}} = 900 \, ^\circ C$, $\dot{\varepsilon} = 1 \, s^{-1}$, b) $T_{\text{def}} = 1000 \, ^\circ C$, $\dot{\varepsilon} = 10 \, s^{-1}$

![Figure 4: Experimental and modelled flow curves of steel S1, hot twisted at: a) $T_{\text{def}} = 900 \, ^\circ C$, $\dot{\varepsilon} = 1 \, s^{-1}$, b) $T_{\text{def}} = 1000 \, ^\circ C$, $\dot{\varepsilon} = 10 \, s^{-1}$](image4)

Slika 4: Eksperimentalna in modelirana krivulja teženja jekla S1, vroče zvijanega pri: a) $T_{\text{def}} = 900 \, ^\circ C$, $\dot{\varepsilon} = 1 \, s^{-1}$, b) $T_{\text{def}} = 1000 \, ^\circ C$, $\dot{\varepsilon} = 10 \, s^{-1}$
values of goal function $\phi = 0.0336$, within the entire range of the temperature and deformations.

In the case of simulative investigations, some constitutive equations were applied, concerning the modelling of the microstructure \cite{13,14}, as well as the kinetics of dynamic recrystallization, developed based on data obtained in plastometric tests for a varying size of the primary austenite grain. As in the rheological model, the kinetics of dynamic recrystallization was taken into account, and it was included in the function of the yield stress, expressed by Equation (1). Thus, in the Sellars model the stress depends on the size of the austenite grain. This model predicts a more accurate flow of the material, particularly in the processes of reiterated deformations in the case of hot rolling.

The rolling of channel iron [240E of type B2 steel was physically simulated based on the parameters of the rolling mill collected in Table 3. These parameters served as the output data for the physical simulation realized on a torsional plastometer in a test of hot torsion, applying the sequential method at a strain rate amounting to about 4.0 s$^{-1}$. The analysed flow curves were compared with the $\sigma - \varepsilon$ curves that were determined in the course of continuous torsion tests in order to determine the effect of global cycles of deformation on the initiation and progress of the activated thermal processes occurring during the intervals between the roll passes. The results of the kinetic investigations, including an analysis of the share of the decay of strain hardening during the interpass times, are presented in the

\begin{table}
\centering
\caption{Optimal coefficients of the rheological model obtained as a result of Simplex optimization concerning the investigated steels with micro-additives}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Steel No. & Rheology – $\sigma_0$ Coefficients & Activation & Goal & & \\
\hline & & $A_0$ & $n_0$ & $\alpha_0$ & energy $Q$ (J/mol) & function $\phi$ & \\
\hline B1 & $4.1200 \cdot 10^{11}$ & 0.0525 & 2.2969 & 278323.4 & 0.0489 & \\
B2 & $1.7800 \cdot 10^{13}$ & 0.1223 & 0.8931 & 330559.2 & 0.0399 & \\
S9 & $2.2300 \cdot 10^8$ & 0.0535 & 36.5915 & 297570.7 & 0.0336 & \\
S1 & $3.7700 \cdot 10^{10}$ & 0.0193 & 21.2038 & 296973.7 & 0.0518 & \\
\hline
\end{tabular}
\end{table}

\begin{table}
\centering
\caption{Characteristics of the process of rolling the profile [240E}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
Stand & Roll & Roll & Working pass & $\varepsilon_s$/% & $\varepsilon_h$/% & Pass input & $S_h$/cm$^2$ & $v_1$/(m/s) & $\varphi$/s$^{-1}$ \\
\hline BD & 1 & 4 & 2.9 & 2.5 & 1150–1180 & 371.0 & 2.52 & 1.32 & \\
Z1 & 2 & 1 & 17.0 & 24.6 & 1140–1170 & 307.9 & 4.54 & 10.30 & \\
Z1 & 3 & 2 & 29.4 & 34.3 & – & 217.3 & 5.35 & 17.56 & \\
Z1 & 4 & 3 & 29.3 & 30.6 & – & 153.6 & 6.12 & 21.86 & \\
Z1 & 5 & 4 & 28.3 & 29.9 & – & 110.2 & 6.84 & 28.36 & \\
Z1 & 6 & 5 & 19.8 & 21.3 & 1050–1080 & 88.4 & 6.89 & 25.54 & \\
Z2 & 7 & 6 & 26.2 & 27.0 & 1020–1055 & 65.2 & 9.01 & 47.80 & \\
Z2 & 8 & 7 & 25.9 & 25.9 & – & 48.3 & 9.09 & 54.36 & \\
Z2 & 9 & 8 & 19.0 & 21.5 & 950–1020 & 39.1 & 8.83 & 52.60 & \\
D1 & 10 & 9 & 15.3 & 12.7 & 960–1050 & 33.9 & 6.97 & 34.57 & \\
D2 & 11 & 10 & 10.0 & 8.8 & 920–950 & 30.5 & 6.93 & 29.16 & \\
\hline
\end{tabular}
\end{table}
diagrams in Figure 5. The deformations in the respective sequential roll passes were found to be less than the critical values required for the initiation of dynamic recrystallization determined by continuous $\sigma - \epsilon$ curves, similar to the case in tests of physical simulations, whereas global deformations in the respective roll passes exceeded the values $\epsilon_{\text{cr}}$. Thus we can assume that in the case of the last roll passes on the flushing stand D1 and D2 the strain hardening is accumulated. This character of the deformation changes determines the occurrence of both static and meta-dynamic recrystallization, and consequently a grain refinement of the investigated steels.

4 CONCLUSIONS

The mathematical modelling of flow stress in the course of high-temperature deformation allows us to obtain the optimal parameters for the thermomechanically controlled processing of the tested structural steels with micro-additives.

Sellars’s rheological model provides good matching of the experimental and theoretical flow curves in the investigated steels, determined on the basis of hot plastometric torsion tests.

A correct model describing the behaviour of HSLA steel type B2 during high-temperature plastic deformation process ensured an accurate simulation of the sequential rolling schedule for a selected economical section of the type [240E].

A physical simulation of the rolling schedule of a channel section [240E] by the means of hot torsion tests ensures the yield stress values $\sigma_y$ in consecutive roll passes and calculates the force and energy parameters in the controlled rolling process.

A kinetic analysis of thermally activated phenomena occurring in the course of high-temperature plastic deformation and interpass times ensures the possibility of shaping the structure of the investigated steels and to determine the mechanical properties of the final products.

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