

MATHEMATICAL MODELLING AND PHYSICAL SIMULATION OF THE HOT PLASTIC DEFORMATION AND RECRYSTALLIZATION OF STEEL WITH MICRO-ADDITIVES

MATEMATIČNO MODELIRANJE IN FIZIKALNA SIMULACIJA VROČE PLASTIČNE PREDELAVE IN REKRISTALIZACIJE JEKLA Z MIKRODODATKI

Elżbieta Kalinowska-Ozgowicz¹, Wojciech Wajda², Wojciech Ozgowicz¹

¹Institute of Engineering Materials and Biomaterials, Silesian University of Technology, ul. Konarskiego 18a, 44-100 Gliwice, Poland

²Institute of Metallurgy and Materials Science, Polish Academy of Science, ul. Reymonta 25, Kraków, Poland
wojciech.ozgowicz@polsl.pl

Prejem rokopisa – received: 2013-10-07; sprejem za objavo – accepted for publication: 2014-02-06

This paper deals with the possibility of optimizing the parameters for a thermomechanical treatment of structural steel with micro-additives, particularly in the processes of the controlled rolling of economical profiles type [240E by means of mathematical modelling of the flow stress (σ_p) and the physical simulation of this process based on the results of investigations obtained by plastometric hot-torsion tests. In the description of the flow stress the rheological model suggested by C. M. Sellars is applied in the paper, in the form $\sigma_p = f(\epsilon, \dot{\epsilon}, T)$. Based on this model, the cause of the experimental and theoretical flow stress-strain curves was verified, applying the minimum of the goal function in order to determine most accurately the matching of the analysed curves of the investigated steels. It was found that the applied rheological model achieves a good matching of the experimental and theoretical curves, whereas a physical simulation permits a complementary verification of the optimal parameters of the controlled rolling of the manufactured products.

Keywords: flow stress, plastometric torsion test, rheological model, physical simulation, thermomechanical controlled processing (TMCP)

Članek obravnava možnost optimizacije parametrov termomehanske obdelave konstrukcijskega jekla z mikrododatki, posebno procese kontroliranega valjanja ekonomskih profilov vrste [240E, z uporabo matematičnega modeliranja napetosti tečenja (σ_p) in fizikalne simulacije tega procesa na osnovi rezultatov raziskav, dobljenih s preizkusi na vročem torzijskem plastometru. V tem članku je bil za opis napetosti tečenja uporabljen reološki model, ki ga je predlagal C. M. Sellars, v obliki $\sigma_p = f(\epsilon, \dot{\epsilon}, T)$. Na podlagi tega modela so bile preizkušene eksperimentalne in teoretične krivulje tečenja – raztezek z uporabo minimuma ciljne funkcije, da bi določili najboljše ujemanje analiziranih krivulj preiskovanih jekel. Ugotovljeno je, da uporabljeni reološki model zagotavlja dobro ujemanje eksperimentalnih in teoretičnih krivulj, medtem ko fizikalna simulacija dopolnjuje preverjanje optimalnih parametrov kontroliranega valjanja proizvodov.

Ključne besede: napetost tečenja, plastometrični torzijski preizkus, reološki model, fizikalna simulacija, termomehansko kontrolirana predelava (TMCP)

1 INTRODUCTION

Mathematical modelling is an essential and economical tool in the application of new techniques for the hot plastic deformation of metals and alloys. It makes it possible not only to reduce the number of technical experiments, but also to assess the interaction of the technological parameters of the investigated processes determining the usability of the final products. Therefore, it integrates, in most cases, the thermal, mechanical and microstructural processes. In the future modelling will constitute the basis of a system for controlling most technological processes. The constructed models will be applied simultaneously in an off-line simulation and on-line control of the processes¹⁻³.

The determination of the parameters of force in thermomechanically controlled processing (TMCP), particularly in the controlled rolling of new profiles, requires a knowledge of the values of the flow stress of the steel under the specific conditions of the deforma-

tion. One of numerous methods for determining these stresses is the hot plastometric torsion test, which also permits a physical simulation of the sequential rolling deformation. Thus, it becomes possible to determine the values of the flow stress in successive roll passes, separated by interpass times in which thermally activated processes occur, removing the results of the strain hardening⁴.

The paper presents possibilities for the mathematical modelling of hot plastic deformation concerning some selected products obtained by rolling structural steels with micro-additives in the range of technological parameters simulating such processes in the plastometric torsion test. Special attention was paid to the verification of the rheological model developed by C. M. Sellars et al.^{5,6} describing the flow stress as a function of the deformation and temperature, as well as strain rate and the effect of dynamic recrystallization, which dominates in the hot deformation for the investigated structural steels with micro-additives of Nb, V, Ti and N.

2 EXPERIMENTAL PROCEDURE

2.1 Materials

The tests were carried out on industrial melts of structural steels with micro-additives, the chemical composition of which is shown in **Table 1**.

Plastometric tests of the steel were performed applying the hot-torsion method. The tested samples had a diameter of 6 mm and a basis length of 10 mm. They were cut out from slabs with dimensions of 200 mm × 220 mm × 5300 mm provided after the hot rolling. By means of continuous torsion, the characteristics of plasticity for the investigated steels were determined in the $\sigma - \epsilon$ system, depending on the temperature of austenitization (1150–1200 °C) and deformation (800–1050 °C) as well as on the strain rate within the range 0.15 s⁻¹ to 10 s⁻¹. The testing was conducted on a torsion plastometer that was designed at the Institute of Iron Metallurgy in Gliwice, Poland. The conversion of torque-twist to stress-strain was generally based on a Fields and Backofen⁷ analysis, commonly quoted in⁸.

2.2 Rheological model

In constitutive equations, actually applied for the purpose of modelling the processes of hot plastic working, the effects of dynamic recovery and dynamic recrystallization are not always conveyed explicitly. The equation in which both components are distinctly separated, elaborated at the University of Sheffield by C. M. Sellars et al.^{5,6}, takes the following form:

$$\sigma = \sigma_0 + (\sigma_{SS(\epsilon)} - \sigma_0) \left[1 - \exp\left(\frac{\epsilon}{\epsilon_r}\right) \right]^{1/2} - R \quad (1)$$

where the respective variables are defined as follows:

$$R = \begin{cases} 0 & \text{for } \epsilon \leq \epsilon_C \\ (\sigma_{SS(\epsilon)} - \sigma_{SS}) \left\{ 1 - \exp\left[-\left(\frac{\epsilon - \epsilon_C}{\epsilon_{xr} - \epsilon_C}\right)^2 \right] \right\} & \text{for } \epsilon > \epsilon_C \end{cases} \quad (1a)$$

$$\sigma_0 = \frac{1}{\alpha_0} \sinh^{-1} \left(\frac{Z}{A_0} \right)^{\frac{1}{n_0}} \quad (1b)$$

$$\sigma_{SS(\epsilon)} = \frac{1}{\alpha_{SS(\epsilon)}} \sinh^{-1} \left(\frac{Z}{A_{SS(\epsilon)}} \right)^{\frac{1}{n_{SS(\epsilon)}}} \quad (1c)$$

$$\sigma_{SS} = \frac{1}{\alpha_{SS}} \sinh^{-1} \left(\frac{Z}{A_{SS}} \right)^{\frac{1}{n_{SS}}} \quad (1d)$$

$$\epsilon_r = 0.31 [q_1 + q_2 (\sigma_{SS(\epsilon)})^2] \quad (1e)$$

$$\epsilon_{xr} - \epsilon_C = \frac{\epsilon_{xs} - \epsilon_C}{1.98} \quad (1f)$$

$$\epsilon_C = C_0 \left(\frac{Z}{\sigma_{SS(\epsilon)}^2} \right)^{N_e} \quad (1g)$$

$$\epsilon_{xs} - \epsilon_C = C_x \left(\frac{Z}{\sigma_{SS(\epsilon)}^2} \right)^{N_x} \quad (1h)$$

$$Z = \dot{\epsilon} \exp\left(\frac{Q_{def}}{RT_{def}}\right) \quad (1i)$$

The variables in Equation (1) are as follows:

- σ_p – flow stress
- σ_0 – the maximum stress when the plastic strain $\epsilon = 0$
- $\sigma_{SS(\epsilon)}$ – the onset of steady-state conditions in the extrapolated curve
- σ_{SS} – the onset of steady-state conditions on the experimental flow stress curve
- ϵ – plastic deformation
- ϵ_C – strain for the onset of dynamic recrystallization
- ϵ_r – the "transient strain constant" and effectively defines the curvature of the flow-stress curve between σ_p and $\sigma_{SS(\epsilon)}$ where the equation saturates
- ϵ_{xr} – the strain required to reach a fixed amount of softening, measured in terms of $\Delta\sigma/\Delta\sigma_s$. This term effectively defines the rate of softening as a result of the dynamic recrystallization
- ϵ_{xs} – the strain at the "onset" of steady state when dynamic recrystallization occurs
- $\dot{\epsilon}$ – strain rate
- Z – Zener – Hollomon parameter
- R – the universal gas constant
- R_x – a term expressing the dynamic recrystallization
- Q_{def} – the activation energy for the deformation
- A, α, n, q, C, N – constants for each characteristic stress σ_p .

Table 1: Chemical composition of the tested steels

Tabela 1: Kemijska sestava uporabljenih jekel

Steel	Concentration of the element in mass fractions, w/%										
	C	Mn	Si	Cr	P	S	Nb	V	Ti	N	Al
B1	0.15	1.03	0.25	–	0.018	0.009	0.017	0.05	–	0.0070	0.040
B2	0.16	1.25	0.32	–	0.023	0.019	0.030	0.01	–	0.0060	0.042
S9	0.38	1.52	0.63	0.56	0.027	0.012	0.030	–	0.11	–	–
S1	0.66	1.02	0.26	0.30	0.029	0.021	–	0.10	–	0.0080	–

The presented model illustrates more distinctly the behaviour of the material in the course of dynamic recrystallization, because it makes it possible to describe the point of deflection ($\varepsilon_{\text{peak}}$) on the curves $\sigma - \varepsilon$ and the procedure of flow stress beginning at the peak strain up to the achievement of the value of the stresses in the steady state (σ_{SS}). The coefficients of this constitutive equation are usually determined based on the results of plastometric tests within a wide range of temperature and strain rates.

This allows us to model various structural materials over a wide range of conditions for hot plastic deformation. The main difficulty in applying this model lies in the large number of parameters, which have to be unambiguously identified. One of the main factors limiting the application of the model simulating the processes of plastic working is the difficulty in determining these coefficients, the values of which depend on the kind of applied material. Nevertheless, Equation (1) has been used in this study for the investigated microalloyed steels.

For the optimization of the parameters of the applied model of C. M. Sellars^{5,6}, describing stress-strain curves, making use of the goal function ϕ in the form:

$$\phi = \sum_{i=1}^{N_p} \sqrt{\frac{1}{N_p} \left(\frac{\sigma_{mi} - \sigma_{ci}}{\sigma_{mi}} \right)^2} \quad (2)$$

where:

σ_m – measured stress

σ_c – calculated stress

N_p – number of measurement points of all the curves in the i -th experiment

For the purpose of the minimum of Equation (2), the Nelder and Mead Simplex algorithm was used. The calculations were performed by means of Scilab's calculation packet.⁹ In results of the optimization for the appropriate parameters of Equation (1) were found.

3 EXPERIMENTAL RESULTS AND DISCUSSION

The results of the mathematical modelling of the process of high-temperature plastic deformation for the investigated micro-alloyed steels allowed us to verify analytically the assumed Equation (1) of the type $\sigma_p = \sigma_p(\varepsilon, \dot{\varepsilon}, T)$ describing the flow stress (σ_p) on the experimental flow-stress curves, making use of the plastometric method for metals and alloys with a low stacking-fault energy (SFE), which in the course of hot deformation display the phenomenon of dynamic recrystallization. Modelling was applied for some selected microalloyed steels of the type HSLA with various contents of the micro-additive Nb (steel B1 and B2), as well as an average-carbon structural steel (0.38 % C) with a binary system of micro-additives of Nb and Ti and a structural rail steel containing about 0.66 % carbon with the micro-additive V (0.10 %). The performed

numerical calculations were based on the results of a hot-torsion test, analysing about 80 stress-strain curves recorded with a wide range of external variables, particularly the temperature and the strain rate. A comparison of the experimental flow curves with those in the model concerning the investigated steels is presented in the diagrams in **Figures 1 to 4**. In order to obtain a universal description of the flow curves, the complete data of measurements $\sigma_p(\varepsilon, \dot{\varepsilon}, T)$ concerning the respective kinds of steel were optimized. The influence of the varying distribution of strains on the radius of the twisted sample, as well as the changes in the temperature at the cross-section of a massive sample have, in the process of optimization, not been taken into account. This might perturb the obtained results, particularly the identified parameters of the rheological model, as has been shown in¹⁰. The elimination of the effect of the homogeneity of deformation and other errors in the measurements ensures the identification of the parameters of the model, applying the inverse method^{11,12}. The paper¹⁰ analyses, however, only the application of this method in the case of various types of hot-compression tests. Future plans will include the application of the inverse method in dealing with the results of hot torsion tests. The numerically determined values of the coefficients assumed in the model, taking into account the Simplex algorithm

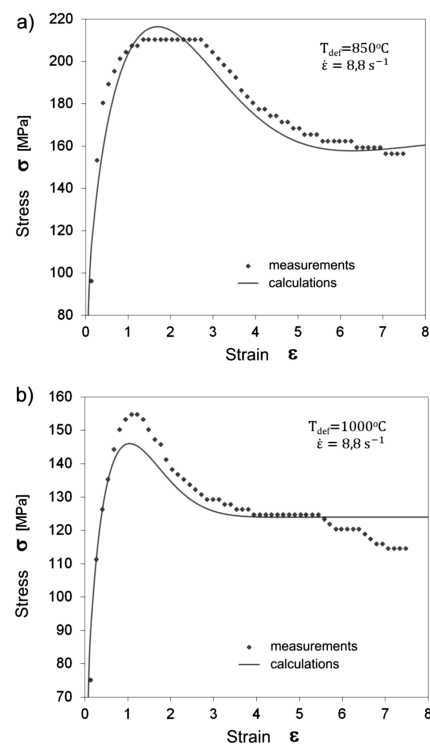


Figure 1: Comparison of the experimental and theoretical flow curves of microalloyed steel B1, hot deformed in compliance with the torsion method at a strain rate of 8.8 s^{-1} and at: a) $T_{\text{def}} = 850 \text{ }^\circ\text{C}$ and b) $T_{\text{def}} = 1000 \text{ }^\circ\text{C}$

Slika 1: Primerjava eksperimentalne in teoretične krivulje tečenja vroče deformiranega mikrolegiranega jekla B1 v primerjavi z metodo torzije pri hitrosti deformacije $8,8 \text{ s}^{-1}$ pri: a) $T_{\text{def}} = 850 \text{ }^\circ\text{C}$ in b) $T_{\text{def}} = 1000 \text{ }^\circ\text{C}$

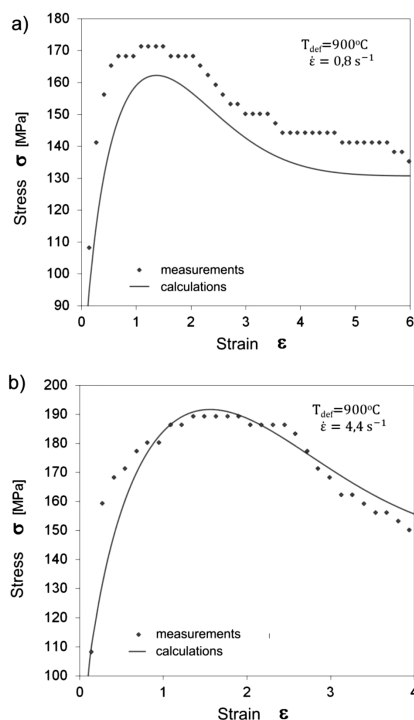


Figure 2: Experimental and modelled flow curves of microalloyed steel B2, hot twisted at $T_{def} = 900\text{ }^{\circ}\text{C}$ and a strain rate of: a) $\dot{\epsilon} = 0.8\text{ s}^{-1}$ and b) $\dot{\epsilon} = 4.45\text{ s}^{-1}$

Slika 2: Eksperimentalna in modelirana krivulja tečenja jekla B2, vroče zvijanega pri $T_{def} = 900\text{ }^{\circ}\text{C}$ in hitrosti deformacije: a) $\dot{\epsilon} = 0,8\text{ s}^{-1}$ in b) $\dot{\epsilon} = 4,45\text{ s}^{-1}$

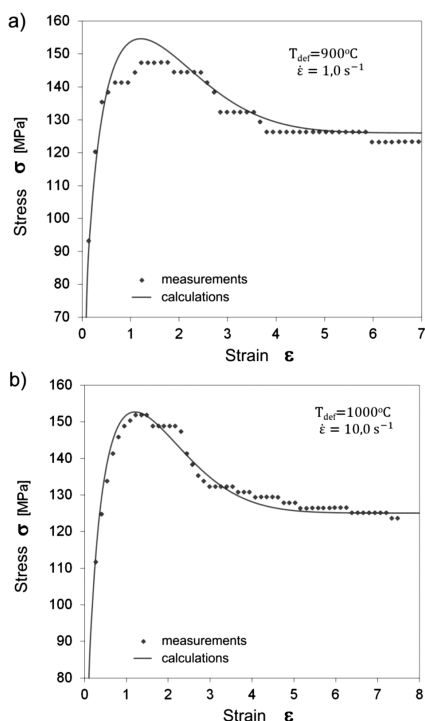


Figure 3: Comparison of experimental and modelled flow curves of steel S9 in a hot torsion test: a) $T_{def} = 900\text{ }^{\circ}\text{C}$, $\dot{\epsilon} = 1\text{ s}^{-1}$, b) $T_{def} = 1000\text{ }^{\circ}\text{C}$, $\dot{\epsilon} = 10\text{ s}^{-1}$

Slika 3: Primerjava eksperimentalne in modelirane krivulje tečenja jekla S9 pri preizkusu vroče torzije: a) $T_{def} = 900\text{ }^{\circ}\text{C}$, $\dot{\epsilon} = 1\text{ s}^{-1}$, b) $T_{def} = 1000\text{ }^{\circ}\text{C}$, $\dot{\epsilon} = 10\text{ s}^{-1}$

concerning the analysed stress-strain curves are shown in **Table 2**, which also contains the values of the energy of activation of the process of deformation and the final values of the goal function ϕ , representing the accuracy of the modelling solution. Due to measurement errors and limitations with the model, different values were obtained for the goal function (Equation (2)) concerning the particular types of investigated steels. However, the median square error of the calculation remains within 2–5 % limits. This proves that the assumed model fits well with the experimental results. The flow curves of the investigated steels indicate principally the characteristic procedure of the continuous dynamic recrystallization. Their analysis permits us to state that this process is essentially influenced by the temperatures (**Figure 1**) and the strain rate (**Figure 2**), as well as the chemical composition of the steel (**Figures 1 to 4**). The effect of these factors concerns, first of all, changes in the value of the maximum flow stress and the stresses in the steady state (σ_{ss}), and also the values of critical deformations (ϵ_f), generally assumed to be in compliance with the criterion of the plasticity of steel in the process of hot deformation. It has been generally assumed that the stresses in the flow of the investigated steels increase with the drop in the temperature and the increasing strain rate. It has also been found that the best compatibility in matching the experimental and theoretical curves is achieved in the case of steel S9 (**Figure 3**), with the

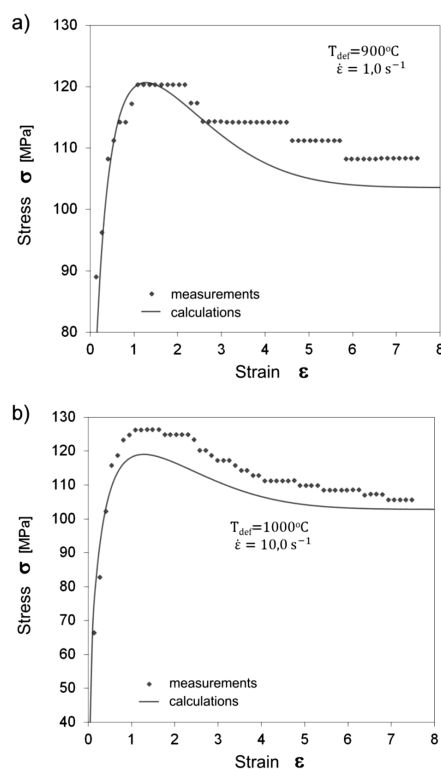


Figure 4: Experimental and modelled flow curves of steel S1, hot twisted at: a) $T_{def} = 900\text{ }^{\circ}\text{C}$, $\dot{\epsilon} = 1\text{ s}^{-1}$, b) $T_{def} = 1000\text{ }^{\circ}\text{C}$, $\dot{\epsilon} = 10\text{ s}^{-1}$

Slika 4: Eksperimentalna in modelirana krivulja tečenja jekla S1, vroče zvijanega pri: a) $T_{def} = 900\text{ }^{\circ}\text{C}$, $\dot{\epsilon} = 1\text{ s}^{-1}$, b) $T_{def} = 1000\text{ }^{\circ}\text{C}$, $\dot{\epsilon} = 10\text{ s}^{-1}$

Table 2: Optimal coefficients of the rheological model obtained as a result of Simplex optimization concerning the investigated steels with micro-additives**Tabela 2:** Optimalni koeficienti reološkega modela, dobljeni kot rezultati optimizacije Simplex preiskovanih jekel z mikroododatki

Steel No.	Rheology – σ_o			Activation energy $Q/(J/mol)$	Goal function ϕ		
	Coefficients						
	A_o	n_o	α_o				
B1	$4.1200 \cdot 10^{11}$	0.0525	2.2969	278323.4	0.0489		
B2	$1.7800 \cdot 10^{13}$	0.1223	0.8931	330559.2	0.0399		
S9	$2.2300 \cdot 10^8$	0.0535	36.5915	297570.7	0.0336		
S1	$3.7700 \cdot 10^{10}$	0.0193	21.2038	296973.7	0.0518		
Steel No.	Rheology – strain hardening and dynamic recovery						
	A_{sse}	n_{sse}	α_{sse}	q_1	q_2		
	B1	$3.19 \cdot 10^{13}$	4.2182	0.0028	0.1668	$0.0067 \cdot 10^{-2}$	
B2	$6.38 \cdot 10^{19}$	4.3216	0.0039	$0.0024 \cdot 10^{-2}$	$0.0075 \cdot 10^{-2}$		
S9	$2.81 \cdot 10^{13}$	5.3884	0.0049	0.6791	$0.0026 \cdot 10^{-2}$		
S1	$1.86 \cdot 10^{11}$	2.4098	0.0203	1.2176	$0.0001 \cdot 10^{-2}$		
Steel No.	Rheology – strain hardening and dynamic recrystallization						
	A_{ss}	n_{ss}	α_{ss}	C_c	N_c	C_x	N_x
	B1	$1.80 \cdot 10^9$	2.4377	0.029	0.000036	0.0244	0.0234
B2	$1.22 \cdot 10^{10}$	2.5560	0.0365	0.0433	0.0079	0.0812	0.1763
S9	$5.99 \cdot 10^8$	3.6426	0.0279	0.0944	0.0378	0.6839	0.0957
S1	$1.21 \cdot 10^9$	3.9124	0.0302	$0.0018 \cdot 10^{-2}$	0.0427	1.4927	0.0670

Table 3: Characteristics of the process of rolling the profile [240E]**Tabela 3:** Značilnosti postopka valjanja profila [240E]

Table of roll passes Profile [240E]								
Slab: 200 mm × 200 mm × 6000 mm; $S_o = 382 \text{ cm}^2$								
Stand	Roll Pass No.	Working pass No.	$\varepsilon_s/\%$	$\varepsilon_h/\%$	Pass input temp. $T/^\circ\text{C}$	S_k/cm^2	$v_1/(\text{m/s})$	φ/s^{-1}
BD	1	4	2.9	2.5	1150–1180	371.0	2.52	1.32
Z1	2	1	17.0	24.6	1140–1170	307.9	4.54	10.30
Z1	3	2	29.4	34.3	–	217.3	5.35	17.56
Z1	4	3	29.3	30.6	–	153.6	6.12	21.86
Z1	5	4	28.3	29.9	–	110.2	6.84	28.36
Z1	6	5	19.8	21.3	1050–1080	88.4	6.89	25.54
Z2	7	6	26.2	27.0	1020–1055	65.2	9.01	47.80
Z2	8	7	25.9	25.9	–	48.3	9.09	54.36
Z2	9	8	19.0	21.5	950–1020	39.1	8.83	52.60
D1	10	9	15.3	12.7	960–1050	33.9	6.97	34.57
D2	11	10	10.0	8.8	920–950	30.5	6.93	29.16

$\varepsilon_s, \varepsilon_h$ indices for the deformation of the rolled channel section [240E]

S_k – running cross-section of the rolled band

v_1 – linear velocity of the rolling

φ – strain rate of the rolled band

values of goal function $\phi = 0.0336$, within the entire range of the temperature and deformations.

In the case of simulative investigations, some constitutive equations were applied, concerning the modelling of the microstructure^{13,14}, as well as the kinetics of dynamic recrystallization, developed based on data obtained in plastometric tests for a varying size of the primary austenite grain. As in the rheological model, the kinetics of dynamic recrystallization was taken into account, and it was included in the function of the yield stress, expressed by Equation (1). Thus, in the Sellars model the stress depends on the size of the austenite grain. This model predicts a more accurate flow of the material, particularly in the processes of reiterated deformations in the case of hot rolling.

The rolling of channel iron [240E of type B2 steel was physically simulated based on the parameters of the rolling mill collected in **Table 3**. These parameters served as the output data for the physical simulation realized on a torsional plastometer in a test of hot torsion, applying the sequential method at a strain rate amounting to about 4.0 s^{-1} . The analysed flow curves were compared with the $\sigma - \varepsilon$ curves that were determined in the course of continuous torsion tests in order to determine the effect of global cycles of deformation on the initiation and progress of the activated thermal processes occurring during the intervals between the roll passes. The results of the kinetic investigations, including an analysis of the share of the decay of strain hardening during the interpass times, are presented in the

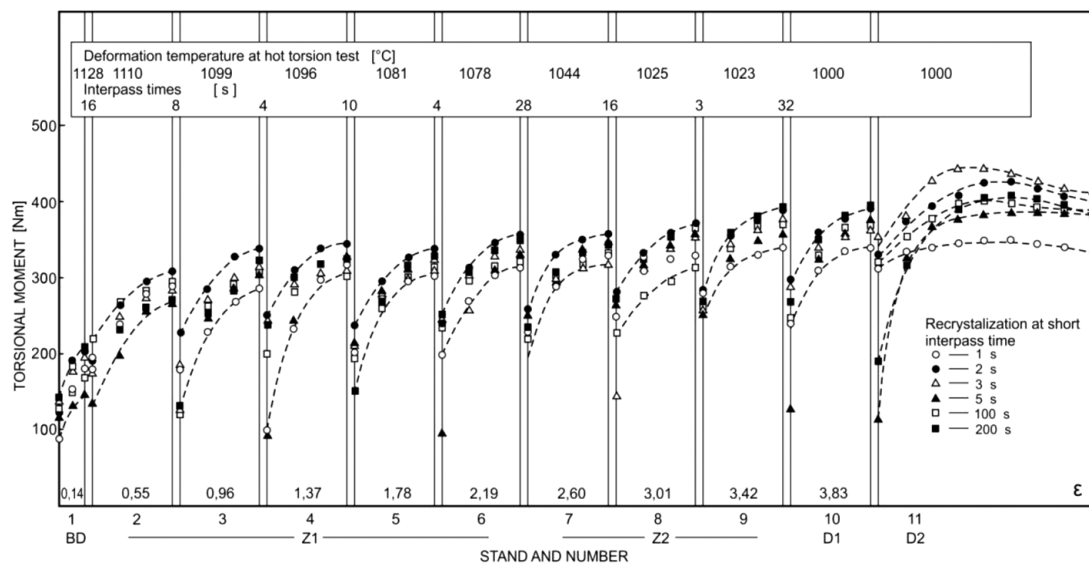


Figure 5: Deformation schedules for channel section type [240E; rolling simulations of microalloyed steel B2
Slika 5: Zaporedje deformacij v kanalskem delu vrste [240E; simulacija valjanja mikrolegiranega jekla B2

diagrams in **Figure 5**. The deformations in the respective sequential roll passes were found to be less than the critical values required for the initiation of dynamic recrystallization determined by continuous $\sigma - \varepsilon$ curves, similar to the case in tests of physical simulations, whereas global deformations in the respective roll passes exceeded the values ε_{cd} . Thus we can assume that in the case of the last roll passes on the flushing stand D1 and D2 the strain hardening is accumulated. This character of the deformation changes determines the occurrence of both static and meta-dynamic recrystallization, and consequently a grain refinement of the investigated steels.

4 CONCLUSIONS

The mathematical modelling of flow stress in the course of high-temperature deformation allows us to obtain the optimal parameters for the thermomechanically controlled processing of the tested structural steels with micro-additives.

Sellars's rheological model provides good matching of the experimental and theoretical flow curves in the investigated steels, determined on the basis of hot plastometric torsion tests.

A correct model describing the behaviour of HSLA steel type B2 during high-temperature plastic deformation process ensured an accurate simulation of the sequential rolling schedule for a selected economical section of the type [240E.

A physical simulation of the rolling schedule of a channel section [240E by the means of hot torsion tests ensures the yield stress values σ_p in consecutive roll

passes and calculates the force and energy parameters in the controlled rolling process.

A kinetic analysis of thermally activated phenomena occurring in the course of high-temperature plastic deformation and interpass times ensures the possibility of shaping the structure of the investigated steels and to determine the mechanical properties of the final products.

5 REFERENCES

- P. D. Hodgson, PhD Thesis, Univ. of Queensland, 1993
- D. Martin, Mater. Sci. Technol., 10 (1994) 10, 855–861
- R. Kuziak, Institute for Ferrous Metallurgy, Gliwice, 2005 (in Polish)
- Z. Kuzminski, A. Nowakowski, M. Gębala, Hutnik – Wiadomości Hutnicze, 8–9 (2006), 394–399
- S. Davenport, N. Silk, C. Sparks, C. M. Sellars, Mater. Sci. Technol., 16 (2000), 539–546
- B. Kowalski, C. M. Sellars, M. Pietrzyk, ISIJ Int., 40 (2000), 1230–1236
- D. S. Fields, W. A. Backofen, 40 (2000) 1, 35
- F. Grosman, E. Hadasik, Silesian University of Technology, Gliwice, 2005 (in Polish)
- <http://www.scilab.org/>
- D. Szeliga, M. Pietrzyk, Arch. Civ. Mech. Eng., 7 (2007) 1, 35
- J. Lenard, M. Pietrzyk, L. Cser, Mathematical and physical simulation of the properties of hot rolled products, Elsevier, Amsterdam 1999
- D. Szyndler, PhD Thesis, AGH, Kraków, 2001 (in Polish)
- R. Kuziak et al., Transaction of the Institute for Ferrous Metallurgy, 1 (2012), 17–23
- E. Kalinowska-Ozgowicz, Structural and mechanical factors of the strengthening and recrystallization of hot plastic deformation of steels with microadditives, Open Access Library, 20 (2013) 2, 1–246