

THE OFF-AXIS BEHAVIOR OF A UNIDIRECTIONAL FIBER-REINFORCED PLASTIC COMPOSITE

ZUNAJOSNO OBNAŠANJE ENOSMERNIH Z VLAKNI OJAČENIH PLASTIČNIH KOMPOZITOV

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Prejem rokopisa – received: 2007-09-20; sprejem za objavo – accepted for publication: 2008-01-10

Numerical and experimental investigations of the off-axis behavior of a unidirectional fiber-reinforced plastic composite consisting of carbon fibers and an epoxy matrix are presented. Static tensile tests of thin carbon-epoxy composite strips with various fiber directions were performed and simulated using a finite-element program. Puck's action-plane concept (failure criterion) and progressive failure analysis were used for the damage prediction of the tested thin strips. A non-linear stress-strain relation was used for modeling the off-axis behavior. The relation between the tensile force and the displacement in the direction of the loading force and the ultimate tensile force, which caused damage and the ultimate displacement of specimens, were measured and compared with the results of the numerical simulation. The fiber directions of the specimens, measured from the direction of loading, were in the interval 0° – 90° .

Good agreement between the experimental results and the numerical simulations was achieved. The non-linear stress-strain relation was verified for the case of a long-fiber carbon-epoxy composite loaded with a static tensile loading force.

Keywords: Composite, PFA, tensile, non-linear, carbon-epoxy, failure

Predstavljeni so rezultati laboratorijske in numerične raziskave zunajosnega obnašanja enosmernih laminatov iz ogljikovih vlaken in epoksi matic. Izvršeni so bili statični preizkusi tankih kompozitnih lamel z različno smerjo vlaken in simulirani z uporabo metode končnih elementov za napoved pojava poškodbe lamel. Pockov model vpliva (kriterij zloma) in progresivna analiza poškodbe sta bila uporabljena za napoved poškodbe preizkusnih tankih lamel. Nelinearen odnos napetost deformacija je bil uporabljen za modeliranje zunaj osnega obnašanja. Določeno je bilo razmerje med natezno silo in premikom v smeri obremenitve in končna natezna sila, ki je povzročila poškodbo ter končni premik in oboje je bilo primerjano z rezultati numerične simulacije. Kot med smerjo vlaken v vzorcih in med smerjo obremenitve je bil med 0° in 90° .

Doseženo je bilo dobro ujemanje med eksperimentalnimi rezultati in rezultati simulacije. Nelinearen odnos napetost deformacija je bil preverjen za primer dolgovlaknatih ogljik-epoksi kompozitov, obremenjenih s statično silo.

Ključne besede: Kompoziti, PFA, natezni preizkusi, nelinearnost, ogljik-epoksi, poškodba

1 INTRODUCTION

New types of materials such as fiber-reinforced plastic (FRP) composites are being used increasingly often. A common model for FRP composites is the transversely isotropic model. This model shows good agreement with experiments in cases when the shear stress (or stresses for the case of a full three-dimensional stress state) does not reach values comparable to the limit of the strength. Therefore, a non-linear stress-strain relation for the FRP composites was developed¹; this takes into account the non-linear behavior of the shear stress in composite materials. It was developed many years ago, but it was not commonly used due to the computational requirements of the non-linear simulations. Also, it is very difficult – and in most cases impossible – to obtain analytical solutions with the use of a non-linear stress-strain relation. Now, however, computational power allows us to solve non-linear problems (mostly with the use of the FEM) in an acceptable time, even on conventional PC configurations, and so it would be useful to apply a more precise stress-strain relation for the modeling of composites. For

this reason the verification of the non-linear stress-strain relation is necessary, and this is the main topic of the paper.

Several papers have already been published by this team of authors in the field of modeling unidirectional composite materials. The identification of the material constants and the failure analysis of unidirectional long-fiber carbon-epoxy composite materials loaded by pure tension force were already performed for three types of specimens (thin composite strips) with different dimensions and fiber directions, with angles 0° , 45° and 90° to the direction of the loading force. The results obtained with the use of the linear stress-strain relation were presented in⁷ and⁸. The results obtained with the use of a non-linear stress-strain relation were presented in⁴. A failure analysis was also performed in the articles mentioned above. The so-called progressive failure analysis, with the use of Puck's action-plane concept (see⁵ or⁶), was carried out in⁷ and⁸, and with the use of Puck's action-plane concept and LaRC04³ the failure criterion was investigated⁴.

Once the material constants were identified for special fiber directions, the verification of the non-linear

stress-strain relation for a unidirectional long-fiber composite can be performed for arbitrary fiber directions.

Static tensile tests of thin, unidirectional carbon-epoxy strips with fiber directions of 0°, 15°, 30°, 45°, 60°, 75° and 90° were performed. The ultimate forces and the force-displacement diagrams were obtained and compared with the results of numerical analyses that were performed with the use of the MSC.Marc 2005r3 FEM software. The progressive failure analysis with the use of Puck's action-plane concept was used for the damage analysis of the strips. The non-linear stress-strain relation and the failure model were implemented in suitable subroutines.

2 NON-LINEAR STRESS-STRAIN RELATION

The linear stress-strain relation (the constitutive relation for a transverse isotropic material) for composite materials can be written in the form ²

$$\begin{pmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{pmatrix} \quad (1)$$

where σ_L and σ_T are the normal stress in the fiber direction (index L – longitudinal) and the normal stress in the direction transverse to the fibers (index T – transverse), and τ_{LT} is the shear stress in the plane of the strip; ε_L and ε_T are the strains in the L, T directions and γ_{LT} is the shear strain in the plane of the layer and matrix C (with the elements C_{ij} dependent on the material constants) is the stiffness matrix.

This form of stress-strain relation is sufficient in cases when the shear stress does not reach values comparable to the limit of the strength. The non-linear behavior of the material is obvious in the latter cases.

Therefore, a non-linear stress-strain relation was developed ¹. It can be written in the form

$$\begin{pmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{pmatrix} + f(\gamma_{LT}) \begin{pmatrix} 0 \\ 0 \\ \gamma_{LT} \end{pmatrix} \quad (2)$$

where $f(\gamma_{LT})$ is the only one real root of the cubic equation

$$y^3 + \frac{3}{S_{66}} y^2 + \left(\frac{3}{S_{66}^2} + \frac{S_{66}}{S_{6666}} \frac{1}{\gamma_{LT}^2} \right) y + \frac{1}{S_{66}^3} = 0 \quad (3)$$

where S_{6666} is the so-called fourth-order compliance coefficient (additional material constant [Pa^{-3}]) and $S_{66} = 1/C_{66}$.

3 FAILURE ANALYSES

Failure criteria are used for the prediction of the composite material's failure. There are several types of failure criteria. Research has shown that a composite

material can be damaged in several modes (matrix cracking, crushing, fiber cracking, and kinking). Therefore, direct mode criteria were developed (Hashin ³, the Puck action-plane concept ^{5,6} or LaRC04 ³).

Puck's action-plane concept was chosen in this work to predict the failure of the examined strips. For a precise description of the criterion see ⁵ and ⁶.

The failure criteria are able to determine the first failure in the model. However, once the propagation of failure is investigated, a progressive failure analysis is needed. The flow chart of the progressive failure analysis (PFA) for a one time pseudo-increment is shown in **Figure 1**. If the failure is recognized in any element for the prescribed loading force, the stiffness matrix of the damaged elements is changed with a dependence on the type of failure (e.g., fiber or matrix). If the model does not contain any damaged elements the loading force increases and the next time pseudo-increment is solved. For further information about PFA see ⁸.

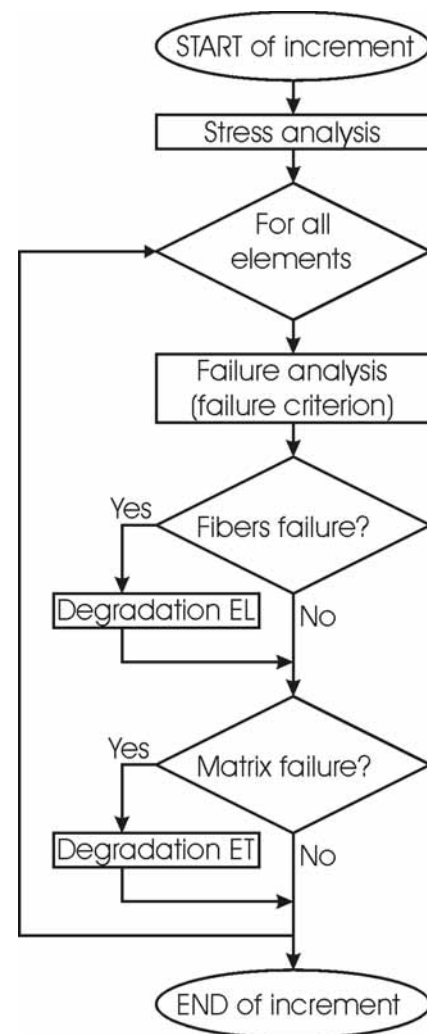


Figure 1: Flow chart of PFA
Slika 1: Potek FPA

4 EXPERIMENTS

The experimental specimens (thin strips) were cut with a water jet from a single, large, unidirectional plate. It was made from four pre-pregs using autoclave technology.

The dimensions of the specimens and the fiber angle and the direction of the loading force are shown in **Figure 2**. If the fiber direction is parallel to the direction of the loading force, the use of aluminum bandages is necessary in order to avoid crushing the specimen between the grips of the testing machine.

As mentioned above, several tests and analyses were already performed on similar types of material (i.e., carbon-epoxy). Previous experiments were performed on the same ZWICK/ROELL Z050 test machine as that used for the experiments in the present work. The set of material constants shown in **Table 1** was identified in ⁴.

Table 1: Previously identified material constants for similar material
Tabela 1: Prej določene konstante za podoben material

Stiffness and Poisson's ratio	
E_L	109.4 GPa
E_T	7.7 GPa
G_{LT}	4.5 GPa
ν_{LT}	0.28
Fourth-order compliance coefficient	
S_{6666}	$1.5 \times 10^{-25} \text{ Pa}^{-3}$
Strength	
X^T	2128 MPa
X^C	1160 MPa
Y^T	44 MPa
Y^C	200 MPa
S^L	52 MPa

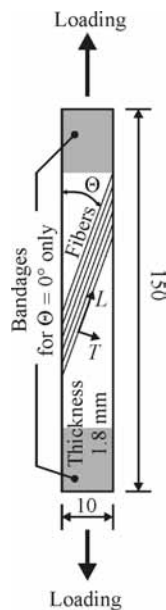


Figure 2: Strips
Slika 2: Preizkusna lamela

In the present work, the experiments were performed for the fiber directions 0°, 15°, 30°, 45°, 60°, 75° and 90°. Ten specimens were tested for each of the fiber directions.

5 RESULTS

The specimens used in the experiments were cut from the plate made from pre-pregs with different thicknesses than the plate used in ^{4,7,8}, and also with slightly different material constants. It is obvious from **Tables 1 and 2** that the fourth-order compliance coefficient S_{6666} is now two times smaller, and that the tensile strength in the transverse direction Y^T and the shear strength S^L are slightly different.

Table 2: Material constants
Tabela 2: Konstante materiala

Stiffness and Poisson's ratio	
E_L	109.4 GPa
E_T	7.7 GPa
G_{LT}	4.5 GPa
ν_{LT}	0.28
Fourth-order compliance coefficient	
S_{6666}	$0.75 \times 10^{-25} \text{ Pa}^{-3}$
Strength	
X^T	2128 MPa
X^C	1160 MPa
Y^T	38 MPa
Y^C	200 MPa
S^L	62 MPa

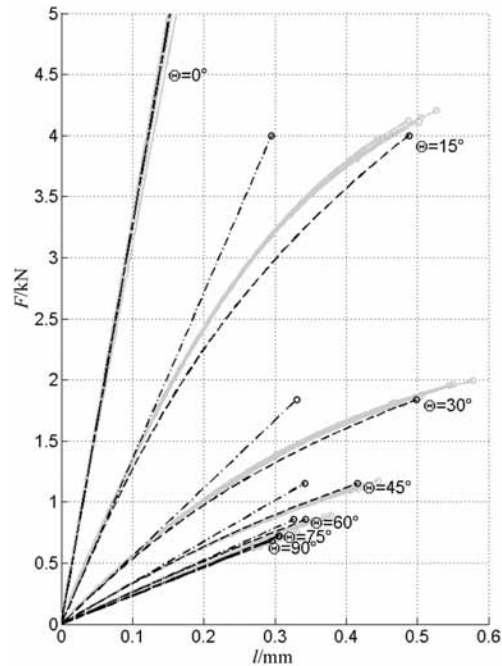


Figure 3: Force-displacement diagram (gray – experiment, dashed – FEM non-linear, dash-and-dot – FEM linear)
Slika 3: Diagram sila-premik (sivo – eksperiment, črtkano – FEM nelinerano, črtno-točkasto – FEM linearno)

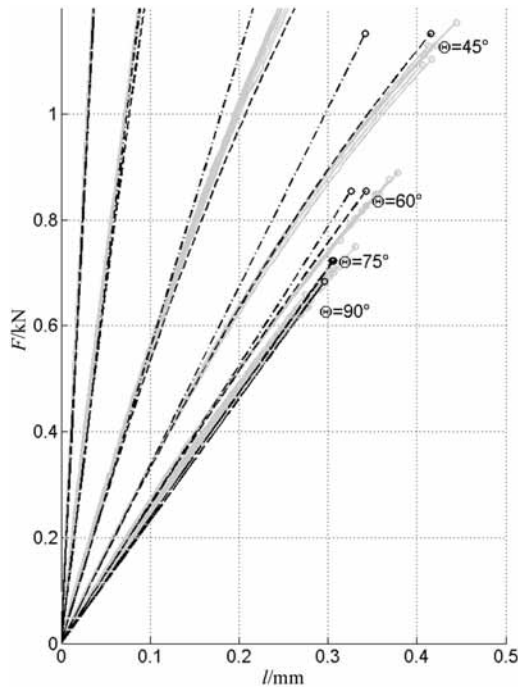


Figure 4: Force-displacement diagram (gray – experiment, dashed – FEM non-linear, dash-and-dot – FEM linear)

Slika 4: Diagram sila-premik (sivo – eksperiment, črtkano – FEM nelinearno, črtnotočkasto- FEM linearno)

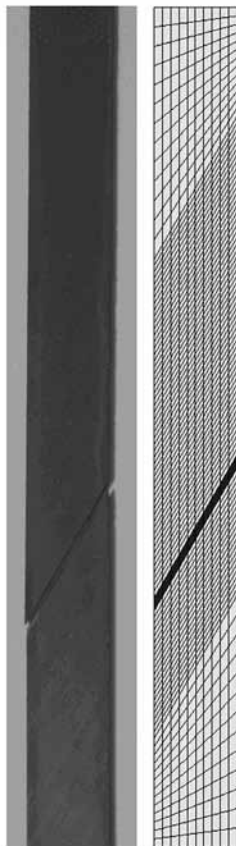


Figure 5: Damaged strip (left – photograph of specimen, right – FEM model)

Slika 5: Poškodovana lamela (levo – lamela, desno – FEM model)

Figure 3 and Figure 4 show force-displacement diagrams for all of the measured fiber angles.

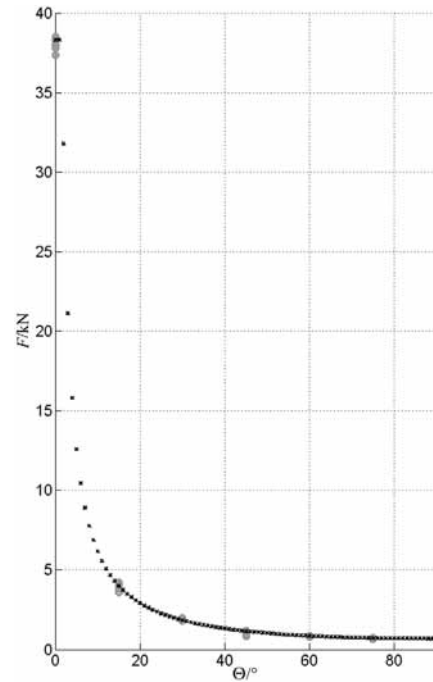


Figure 6: Ultimate force as a function of fiber angle Θ (gray circle – experiment, black dot and cross – linear and non-linear FEM)

Slika 6: Končna sila v odvisnosti od kota vlaken Θ (sivi krogci – eksperiment, črne točke in križci – linearni in nelinearni FEM)

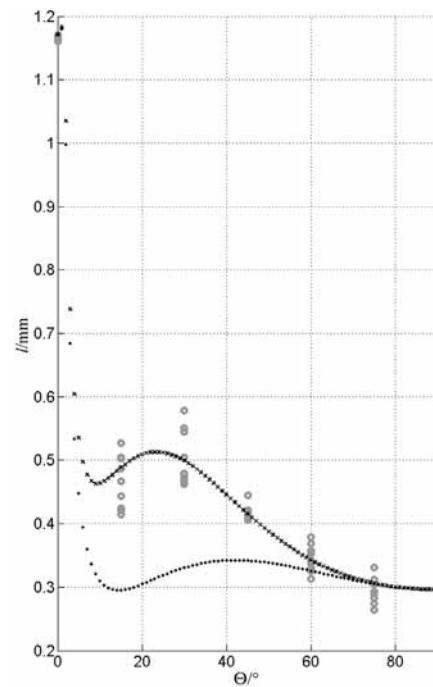


Figure 7: Ultimate displacement of the strips as a function of the fiber angle Θ (gray circle – experiment, cross – non-linear FEM, dot – linear FEM)

Slika 7: Končni premik lamel v odvisnosti in kota vlaken Θ (sivi krogci – eksperiment, križci – nelinearni FEM, točke – linearni FEM)

Figure 5 shows the damaged strip with a fiber angle of $\Theta = 30^\circ$. On the left-hand side is the real damaged material, and on the right-hand side is the damaged FEM model (the black elements are damaged – tensile failure mode of matrix). **Figure 6** shows the ultimate force as a function of the fiber angle.

As shown in **Figure 6**, the ultimate tensile force is a strictly decreasing function of the fiber angle. In contrast to this, the ultimate displacement of the strips is not. As shown in **Figure 7**, there is a local maximum of the function around $\Theta = 23^\circ$. The difference between the linear and the non-linear stress-strain relation is obvious from **Figure 7**.

6 CONCLUSION

The non-linear stress-strain relation was verified by a simple tensile test for the case of a unidirectional long-fiber carbon-epoxy composite for the full range of fiber angles, 0° – 90° .

The numerical analyses were performed with the use of non-linear and linear stress-strain relations in order to compare the influence of nonlinearity. The numerical analysis combining the non-linear stress-strain relation with the progressive failure analysis using Puck's action-plane concept shows good agreement with the experiments.

Further research will focus on the non-linear behavior of laminated composites.

Acknowledgements: The work has been supported by the grant projects GA AS CR no. A200760611 and GACR no. 101/07/P059.

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