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NUMERICAL SOLUTION OF HOT SHAPE ROLLING OF STEEL

NUMERIČNA REŠITEV VROČEGA VALJANJA JEKLA

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The modeling of hot shape rolling of steel is represented by using a meshless method. The physical model consists of coupled thermal and mechanical models. Both models are numerically solved by using a strong formulation. The material is assumed to behave ideally plastic. The model decomposes the 3D geometry of the steel billet into a traveling 2D cross section which lets us analyze the large shape reductions by a sequence of small steps. A uniform velocity over each of the cross-sections is assumed. The meshless method, based on collocation with radial basis functions is used to solve the thermo-mechanical problem. The node distribution is calculatedby elliptic node generation at each deformation step to the new form of the billet. The solution is calculated in terms of temperatures and displacements at each node. Preliminary numerical examples for the new rolling mill in Štore Steel are shown.

Keywords: steel, hot rolling, radial basis functions, meshless numerical method

Modeliranje vročega valjanja je predstavljeno z uporabo brezmrežne numerične metode. Fizikalni model je sestavljen iz sklopljenega termičnega in mehanskega modela. Oba sta numerčno rešena z uporabo močne formulacije. Predpostavljamo, da se material vede idealno plastično. V modelu razstavimo 3D-geometrijo jeklene gredice v premikajoč 2D-prerez, ki omogoča analizo velikih sprememb oblike v majhnih korakih. Predpostavimo uniformno hitrost preko vsakega prereza. Za rešitev termo-mehanskega problema je uporabljena brezmrežna numerična metoda, ki temelji na kolokaciji z radialnimi baznimi funkcijami. Distribucijo diskretizacijskih točk smo za vsako novo obliko prereza gredice izračunali na podlagi eliptičnega generatorja diskretizacijskih točk. Rešitev je podana kot temperatura in premik v vsaki točki. Prikazani so preliminarni numerični primeri za novo valjarsko progo v podjetju Štore Steel.

Ključne besede: jeklo, vroče valjanje, radialne bazne funkcije, brezmrežna numerična metoda

1 INTRODUCTION

The main aim of this paper is elaboration of the coupled thermo-mechanical computational model developed for hot shape rolling of steel. The output of the thermal model is the temperature field and mechanical model the displacement (deformation). Shape rolling is a 3D process, however it is analyzed with 2D imaginary slices which is denoted as a slice model. The coordinate system of a 2D slice is based on Langrangian description where the slice travels across the rolling contact. The third axis, the rolling direction, is based on the Eularian description where there is a constant inflow and outflow of steel through the rolling direction. This is considered as a mixed Eularian-Langrangian model. It was discussed previously by many authors [1,2].

In many publications of rolling Finite Element Method (FEM) was used which is based on a mesh. A novel numerical method used in this paper to solve the involved partial differential equations is the Local Radial Basis Function Collocation Method (LRBFCM). This is a completely meshless procedure. LRBFCM has been recently used in highly sophisticated simulations like multi-scale solidification modeling [3], convection driven melting of anisotropic metals [4], continuous casting of steel [5]. This paper is organized in a way that, first the thermal model and afterwards the mechanical model are developed. Overall it becomes a coupled thermo mechanical model. The flow chart of the process is shown in **Figure 1**.

2 THERMAL MODEL

The thermal model of the shape rolling process is aimed to calculate the temperature field of the steel slab during the rolling process. The three dimensional domain Ω^{3D} with boundary Γ^{3D} is considered. The solution procedure is based on Cartesian coordinate system with axes x, y, z. Slices coincide with coordinates and the rolling direction is z. The steady state temperature distribution in the rolled product is defined through the following equation,

$$\nabla \cdot (\rho c_p \mathbf{v} T) = \nabla \cdot (k \nabla T) + S \; ; \; \mathbf{p} \in \Omega^{3D} \quad (x,y,z) \qquad (1)$$

Since we analyze the process with 2D slices perpendicular to the rolling direction and assume thata uniform velocity over the slices (homogenous compression) takes place. The Equation (1) can be transferred into

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + S \; ; \; \mathbf{p} \in \Omega^{2D} \quad (x,y)$$
$$p_z(z,t) = \int v dt = v_{\mathrm{entry}} A_{\mathrm{entry}} \int \frac{1}{A(z)} \mathrm{d}t \qquad (2)$$

with $\mathbf{p}$, ρ, t, c_p, T, k, v_{entry}, A_{entry}, $(A)z$ and S standing for position vector, density, time, specific heat, temperature, thermal conductivity, entry speed of billet, entry cross sectional area, instant cross sectional area and internal heat generation due to plastic deformation. It is assumed in the slice model that the heat transport takes place only in the direction perpendicular to the rolling direction and that the homogenous deformation takes place. The Neumann boundary condition on the part of the boundary denoted as Γ^N, Robin boundary condition on the part of the boundary denoted as Γ^R are taken into consideration ($\Gamma = \Gamma^N \cup \Gamma^R$) which are described below,

$$-k\nabla T(\mathbf{p})\cdot \mathbf{n}_\Gamma = -k\frac{\partial T(\mathbf{p})}{\partial \mathbf{n}_\Gamma} = q \; ; \; \mathbf{p} \in \Gamma^N \tag{3}$$

$$-k\nabla T(\mathbf{p})\cdot \mathbf{n}_\Gamma = -k\frac{\partial T(\mathbf{p})}{\partial \mathbf{n}_\Gamma} = h\left[T(\mathbf{p}) - T_\Gamma^R(\mathbf{p})\right] \; ; \; \mathbf{p} \in \Gamma^R \tag{4}$$

The N_Ω nodes at the domain and N_Γ nodes at the boundary are used to discretize the temperature in LRBFCM where for each node $\mathbf{p}_n = \{p_x, p_y\}^T$. For each node there is a defined influence domain with N_ω neighboring nodes. For each influence domain a radial basis function in terms of multiquadric is written

$$y_i = \sqrt{(p_x - p_{xn})^2 / x_{\max} + (p_y - p_{yn})^2 / y_{\max} + c^2}$$

The temperature can now be interpolated as

$$T = \sum_{n=1}^{N_\omega} \psi_n \alpha_n$$

with the collocation coefficients to be determined. The main equation in 2D can be rewritten by using the explicit time stepping,

$$\rho c_p \frac{T_{i+1} - T_i}{\Delta t} = \nabla k_i \cdot \sum_{n=1}^{N_\omega} (\nabla \psi_n)\alpha_n + \left(k_i \cdot \sum_{n=1}^{N_\omega} (\nabla \psi_n)\alpha_n\right) + S \; ; \; \mathbf{p} \in \Omega^{2D} \tag{5}$$

3 MECHANICAL MODEL

A strong form is chosen here for analysis due to its compatibility with LRBFCM. Adomain Ω with boundary Γ, $\Gamma = \Gamma^U \cup \Gamma^T$ is considered where Γ^U is the essential and Γ^T represents the natural boundary conditions. The strong formulation of the static metal deformation problems is:

$$\mathbf{L}^T \sigma + \mathbf{b} = \mathbf{0} \tag{6}$$

In the calculations, in order to avoid complications of a 3D solution, the slab is analyzed, compatible with the thermal model, with imaginary traveling 2D slices that are perpendicular to the rolling direction. For a 2D slice method, $\mathbf{L}$ is the 3×2 derivative matrix with elements $L_{11} = \partial/\partial p_x$, $L_{12} = 0$, $L_{21} = 0$, $L_{22} = \partial/\partial p_y$, $L_{31} = \partial/\partial p_y$ and $L_{32} = \partial/\partial p_y$, $\boldsymbol{\sigma} = [\sigma_x, \sigma_y, \sigma_{xy}]^T$ is the stress vector, and $\mathbf{b} = [b_x, b_y]^T$ is the body force. At the essential boundary Γ^U

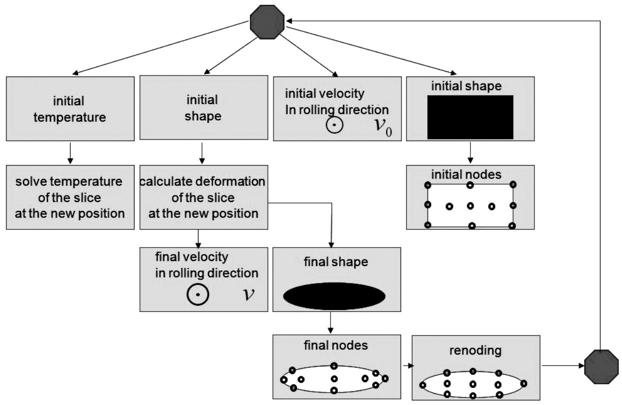

Figure 1: Flow chat of the coupled thermo mechanical model.
Slika 1: Bločni diagram sklopljenega termo-mehanskega modela

$$\mathbf{u}(\mathbf{p}) = \bar{\mathbf{u}}(\mathbf{p}) \; ; \; \mathbf{p} \in \Gamma^U \tag{7}$$

where $\mathbf{u}(\mathbf{p})$ is displacement vector and $\bar{\mathbf{u}}(\mathbf{p})$ is the prescribed displacement vector. At the naturalboundary condition Γ^T

$$\mathbf{N}^T \boldsymbol{\sigma} = \bar{\boldsymbol{\tau}} \; ; \; \mathbf{p} \in \Gamma^T \tag{8}$$

is valid, where $\bar{\boldsymbol{\tau}}$ is the prescribed surface traction $\bar{\boldsymbol{\tau}} = [\bar{\tau}_x, \bar{\tau}_y]^T$, $\mathbf{N}$ is the 2×3 matrix of direction cosines of the normal direction at the boundary which can be defined as $N_{11} = N_{32} = n_x$, $N_{12} = N_{21} = 0$, $N_{31} = N_{22} = n_y$ (n_x,n_y) represent correlation of the normal at the boundary). In a 2D system the equations for mechanical model can be written as,

$$\frac{\partial \sigma_x}{\partial p_x} + \frac{\partial \sigma_{xy}}{\partial p_y} + b_x = 0, \quad \frac{\partial \sigma_y}{\partial p_y} + \frac{\partial \sigma_{xy}}{\partial p_x} + b_y = 0 \tag{9, 10}$$

The discretization is made in terms of displacement on x and y axes for each slice,

$$u_x(\mathbf{p}) = \sum_{n=1}^{N_\omega} \psi_n(\mathbf{p})\alpha_{xn}, \; u_y(\mathbf{p}) = \sum_{n=1}^{N_\omega} \psi_n(\mathbf{p})\alpha_{yn} \tag{11, 12}$$

Since the strain vector $\varepsilon = [\varepsilon_x, \varepsilon_y, \varepsilon_{xy}]^T$ can be written in terms of displacement as $\boldsymbol{\varepsilon} = \mathbf{Lu}$, the strain vector σ can be expressed as a stress vector by using 6×6 stiffness matrix $\mathbf{C}$ which depends on the material characteristic assumption such as elastic, elastic-plastic or ideally plastic.

$$\sum_{n=1}^{N_\omega} \alpha_{xn}\left[C_{11}\frac{\partial^2 \psi_n}{\partial p_x^2} + C_{33}\frac{\partial^2 \psi_n}{\partial p_y^2} + (C_{13} + C_{31})\frac{\partial^2 \psi_n}{\partial p_y \partial p_x}\right] + \sum_{n=1}^{N_\omega} \alpha_{yn}\left[(C_{12} + C_{33})\frac{\partial^2 \psi_n}{\partial p_y \partial p_x} + C_{13}\frac{\partial^2 \psi_n}{\partial p_x^2} + C_{32}\frac{\partial^2 \psi_n}{\partial p_y^2}\right] + b_x = 0 \tag{13}$$

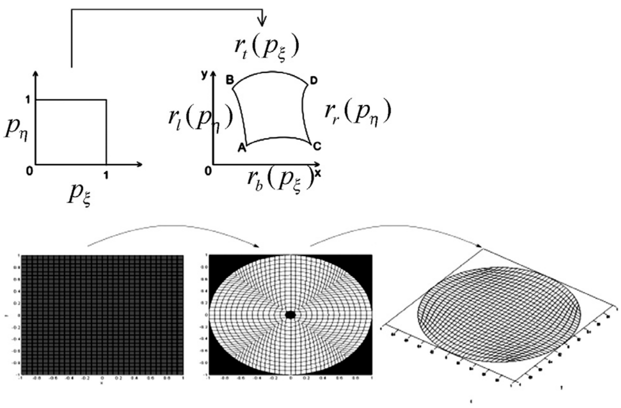

Figure 2: Transformation from computational domain to physical domain (left), TFI and nodes displacement through ENG (right). The collocation points are put on the intersection of grid lines.

Slika 2: Transformacija izračunskega območja v fizično območje (levo) TFI in premik točk preko ENG (desno). Kolokacijske točke so postavljene v prerez mrežnih linij.

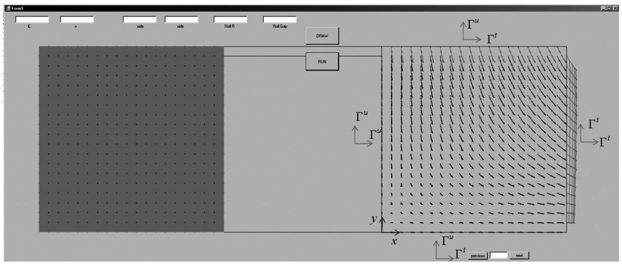

Figure 3: Simulation of flat rolled (180 × 180) mm cross sectioned 16MnCrS5 steel at 1100 °C with Young's modulus E = 97362.21 MPa and Poisson's ratio v = 0.35678. The total reduction is 16.66 % and preliminary analyzed with 5 slices by using elastic stiffness matrix [7]. Arrows represents the displacement vector for each slice. The exit speed is equal to 1.14389 times the entry speed of the billet. Due to symmetry only the top right part of the billet is considered.

Slika 3: Simulacija prereza (180 × 180) mm ploščatega valjanja za jeklo16MnCrS5 pri 1100 °C z Youngovim modulom E = 97362.21 MPa in Poissonovim razmerjem v = 0.35678. Skupno zmanjšanje je 16,66 % in predhodno analizirano s 5 rezinami z uporabo elastične togostne matrike [7]. Puščice pomenijo vektor premika za vsako rezino. Izhodna hitrost je enaka 1.14389-kratniku vhodne hitrosti gredice. Zaradi simetrije je upoštevana samo zgornja polovica gredice.

$$\sum_{n=1}^{N_\omega} \alpha_{xn}\left[(C_{21}+C_{33})\frac{\partial^2\psi_n}{\partial p_y \partial p_x}+C_{31}\frac{\partial^2\psi_n}{\partial p_x^2}+C_{23}\frac{\partial^2\psi_n}{\partial p_y^2}\right]+ \\ +\sum_{n=1}^{N_\omega} \alpha_{yn}\left[C_{22}\frac{\partial^2\psi_n}{\partial p_y^2}+(C_{23}+C_{32})\frac{\partial^2\psi_n}{\partial p_y \partial p_x}+C_{33}\frac{\partial^2\psi_n}{\partial p_x^2}\right]+b_y=0 \tag{14}$$

4 TRANSFINITE INTERPOLATION (TFI)

This technique is used to generate initial grid which is confirming to the geometry encountered in different stages of plate and shape rolling. Suppose that there exists a transformation $\mathbf{r}(p_\xi, p_\eta) = \{p_x(p_\xi, p_\eta), p_y(p_\xi, p_\eta)\}^T$ which maps the unit square, $0 < p_\xi < 1$, $0 < p_\eta < 1$ in the computational domain onto the interior of the region ABCD in the physical domain such that the edges p_ξ = 0,1 map to the boundaries AB, CD and the edges p_η = 0,1 are mapped to the boundaries AC, BD. The transformation is used for this purpose is defined as

$$\mathbf{r}(p_\xi,p_\eta) = (1-p_\eta)\mathbf{r}_l(p_\eta) + \xi\mathbf{r}_r(p_\eta)\mathbf{r}_b(p_\xi) + \\ p_\eta\mathbf{r}_t(p_\xi) - (1-p_\xi)(1-p_\eta)\mathbf{r}_b(0) - (1-p_\xi)p_\eta\mathbf{r}_t(0) - \\ (1-p_\eta)p_\xi\mathbf{r}_b(1) - p_\xi p_\eta\mathbf{r}_t(1) \tag{15}$$

Where $\mathbf{r_b}$, $\mathbf{r_t}$, $\mathbf{r_l}$, $\mathbf{r_r}$ represent the values at the bottom, top, left and right edges respectively. The initial grid is refined through ENG [6]. **Figure 2** shows initial node generation through TFI and its correlation with ENG.

5 CONCLUSION

In this paper the thermal and mechanical formulations are given for hot shape rolling. The numerical method for the solution of the problem is based on meshfree LRBFCM. The preliminary result of mechanical model for elastic case is presented in **Figure 3**. The future work will include plastic deformation in a sequence of 10 rolling stands as recently installed in Štore –Steel Company.

6 REFERENCES

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