

# NUMERICAL STUDY OF RAYLEIGH-BÉNARD NATURAL-CONVECTION HEAT-TRANSFER CHARACTERISTICS OF WATER-BASED Au NANOFLUIDS

## NUMERIČNA ANALIZA PRENOSA TOPLOTE NANOTEKOČIN VODA-Au V RAZMERAH REYLEIGH-BÉNARDOVE NARAVNE KONVEKCIJE

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The present work deals with the natural convection in a square cavity filled with a water-based Au nanofluid. The cavity is heated from the lower and cooled from the adjacent wall, while the other two walls are adiabatic. The governing differential equations have been solved with the standard finite volume method and the hydrodynamic and thermal fields have been coupled using the Boussinesq approximation. The main objective of this study is to investigate the influence of the nanoparticles' volume fraction on the heat-transfer characteristics of Au nanofluids at a given base-fluid (i.e., water) Rayleigh number  $Ra_{bf}$ . Accurate results are presented over a wide range of the base-fluid Rayleigh numbers ( $10^2 \leq Ra_{bf} \leq 10^5$ ) and the volume fraction of Au nanoparticles ( $0\% \leq \varphi \leq 10\%$ ). It is shown that adding nanoparticles to the base fluid delays the onset of convection. Contrary to what is argued by many authors, we show, with numerical simulations, that the use of nanofluids can reduce the heat transfer instead of increasing it.

Keywords: Rayleigh-Bénard natural convection, water-Au nanofluid, heat transfer, numerical modelling

V prispevku obravnavamo naravno konvekcijo v kvadratni kotanji, napoljeni z nanotekočino voda-Au. Kotanja je bila greta s spodnje in hlajena s priležne zgornje stene, preostali dve steni sta bili adiabatni. Vodilne diferencialne enačbe smo reševali s standardno metodo končnih prostornin. Hidrodinamično in temperaturno polje sta bila sklopljena z uporabo Boussinesqove aproksimacije. Glavni cilj prispevka je raziskati vpliv prostorninskega deleža nanodelcev na značilnosti prenosa toplote Au-nanotekočine pri podani vrednosti Rayleighjevega števila nosilne tekočine (vode)  $Ra_{bf}$ . Natančni rezultati so predstavljeni za široko območje vrednosti Rayleighjevega števila nosilne tekočine ( $10^2 \leq Ra_{bf} \leq 10^5$ ) in prostorninskega deleža Au-nanodelcev ( $0\% \leq \varphi \leq 10\%$ ). Pokazali smo, da dodajanje nanodelcev v nosilno tekočino zakasni začetek naravne konvekcije. V nasprotju s trditvami mnogih avtorjev smo z numeričnimi simulacijami pokazali, da lahko uporaba nanodelcev prenos toplote zmanjša in ne poveča.

Ključne besede: Rayleigh-Bénardova naravna konvekcija, nanotekočina voda-Au, prenos toplote, numerično modeliranje

## 1 INTRODUCTION

Buoyancy-induced flow together with the associated heat transfer is an important phenomenon found in many engineering applications (e.g., selective laser melting process<sup>1</sup>, cooling of electronic devices<sup>2</sup>). An enhancement of heat transfer in such systems is crucial from the energy-saving point of view. In recent years, nanosized particles dispersed in a base fluid, known as nanofluid, has been used and researched extensively to enhance the heat transfer. The presence of nanoparticles shows an unquestionable heat-transfer enhancement in forced convection applications<sup>3</sup>. However, with respect to the buoyancy-driven flow, there is still a dispute on the effect of nanoparticles on the heat-transfer enhancement.

Several researchers have been focused on the numerical modelling of buoyancy-induced flows. Recent numerical studies by Ternik et al.<sup>4</sup>, Ternik and Rudolf<sup>5</sup>,

Oztop et al.<sup>6</sup> and Abu-Nada and Oztop<sup>7</sup> illustrated that the suspended nanoparticles substantially increase the heat-transfer rate for any given Rayleigh number. In addition, they showed that the heat-transfer rate in water-based nanofluids increases with an increasing volume fraction of  $Al_2O_3$ , Cu,  $TiO_2$  and Au nanoparticles.

On the other hand, an apparently paradoxical behaviour of the heat-transfer deterioration was observed in many experimental studies<sup>8-10</sup>. For example, Putra et al.<sup>8</sup> reported that a presence of  $Al_2O_3$  nanoparticles in a base fluid reduces the natural convective heat transfer. However, they did not clearly explain why the natural convective heat transfer is decreased with an increase in the volume fraction of nanoparticles.

The above review of the existing literature shows that the problem of natural convection in a bottom-heated horizontal cavity filled with a nanofluid is an issue still

far from being completely solved. Framed in this general background, the purpose of the present study is to examine the effect of adding Au nanoparticles to the base fluid at the conduction and convection heat-transfer rates in a square cavity heated from below (Rayleigh–Bénard configuration) over a range of base-fluid Rayleigh numbers  $10^2 \leq Ra_{bf} \leq 10^5$  and volume fractions  $0 \% \leq \varphi \leq 10 \%$ .

## 2 NUMERICAL MODELLING

The standard finite-volume method, successfully used in many recent studies,<sup>11–13</sup> is used to solve the coupled conservation equations of mass, momentum and energy. In this framework, a second-order central differencing scheme is used for the diffusive terms and a second-order upwind scheme for the convective terms. Coupling of the pressure and velocity is achieved using the SIMPLE algorithm. The convergence criteria were set to  $10^{-9}$  for all the relative (scaled) residuals.

### 2.1 Governing equations

For the present study, a steady-state flow of an incompressible water-based Au nanofluid is considered. It is assumed that both the fluid phase and nanoparticles are in thermal equilibrium. Except for the density, the properties of the nanoparticles and fluid (presented in **Table 1**) are taken to be constant. The Boussinesq approximation is invoked for the nanofluid properties to relate density changes to temperature changes, and to couple the temperature field with the velocity field.

The governing equations (mass, momentum and energy conservation) of such a flow are:<sup>4,5</sup>

$$\frac{\partial v_i}{\partial x_i} = 0 \tag{1}$$

$$\rho_{nf} v_j \frac{\partial v_i}{\partial x_j} - \frac{\partial}{\partial x_j} \left( \eta_{nf} \frac{\partial v_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + (\rho\beta)_{nf} g(T - T_C) + \frac{\partial}{\partial x_j} \left( \eta_{nf} \frac{\partial v_j}{\partial x_i} \right) \tag{2}$$

$$(\rho c_p)_{nf} v_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left( c_{nf} \frac{\partial T}{\partial x_j} \right) \tag{3}$$

where the cold-wall temperature  $T_C$  is taken to be the reference temperature for evaluating the buoyancy term  $(\rho\beta)_{nf}g(T - T_C)$  in the momentum conservation equation.

**Table 1:** Thermo-physical properties of the Au nanofluid<sup>4,5</sup>

**Tabela 1:** Toplotno-fizikalne lastnosti Au-nanotekočine<sup>4,5</sup>

	$\eta$ (Pa s)	$\rho$ (kg/m <sup>3</sup> )	$c_p$ (J/kg K)	$k$ (W/m K)	$\beta$ (1/K)
Pure water	$1.003 \times 10^{-3}$	997.1	4179	0.613	$2.1 \times 10^{-4}$
Au	/	19320	128.8	314.4	$1.416 \times 10^{-7}$

Relationships between the properties of the nanofluid ( $nf$ ) and those of the base fluid ( $bf$ ) and pure solid ( $s$ ) are given with the following empirical models<sup>4,5</sup>:

- Dynamic viscosity:

$$\eta_{nf} = \frac{\eta_{bf}}{(1-\varphi)^{2.5}}$$

- Density:

$$\rho_{nf} = (1-\varphi)\rho_{bf} + \varphi\rho_s$$

- Thermal expansion:

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_{bf} + \varphi(\rho\beta)_s$$

- Heat capacitance:

$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_{bf} + \varphi(\rho\beta)_s$$

- Thermal conductivity:

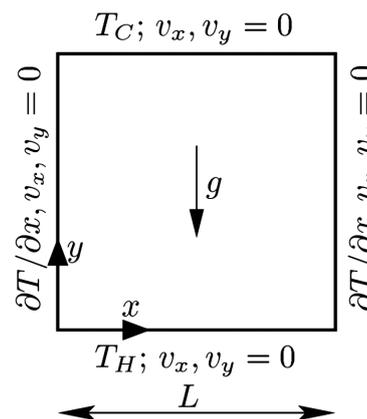
$$k_{nf} = k_{bf} \frac{k_s + 2k_{bf} - 2\varphi(k_{bf} - k_s)}{k_s + 2k_{bf} + \varphi(k_{bf} - k_s)}$$

### 2.2 Geometry and boundary conditions

The simulation domain is shown schematically in **Figure 1**. The two horizontal walls of a square enclosure are kept at different constant temperatures ( $T_H > T_C$ ), whereas the other boundaries are considered to be adiabatic. Both velocity components (i.e.,  $v_x$  and  $v_y$ ) are identically zero on each boundary because of the no-slip condition and the impenetrability of the rigid boundaries.

In the present study, the heat-transfer characteristics are presented in terms of the mean Nusselt number:

$$\overline{Nu} = \frac{1}{L} \int_0^L Nu(x) dx \tag{4}$$



**Figure 1:** Schematic diagrams of the simulation domain  
**Slika 1:** Shematski prikaz območja simulacije

and the ratio of the nanofluid heat-transfer rate to the base-fluid one:

$$\frac{Q_{nf}}{Q_{bf}} = \frac{k_{nf} \overline{Nu_{nf}}}{k_{bf} \overline{Nu_{bf}}} = \frac{h_{nf}}{k_{bf}} \quad (5)$$

where  $h_{nf}$  and  $h_{bf}$  are the convection heat-transfer coefficients of the nanofluid and the base fluid.

In order to investigate the influence of volume fraction  $\phi$  on the heat-transfer characteristics, the Rayleigh ( $Ra_{nf}$ ) and the Prandtl numbers ( $Pr_{nf}$ ) for the nanofluids are expressed as follows:

$$Ra_{nf} = \frac{(\rho\beta)_{nf} k_{bf} (\rho c_p)_{nf} \eta_{bf}}{(\rho\beta)_{bf} k_{nf} (\rho c_p)_{bf} \eta_{nf}} Ra_{bf} \quad (6)$$

$$Pr_{nf} = \frac{\eta_{nf} c_{p,nf} k_{bf}}{\eta_{bf} c_{p,bf} k_{nf}} Pr_{bf}$$

Using equation (6) we show that  $Ra_{nf} < Ra_{bf}$  (Figure 2a) and  $Pr_{nf} < Pr_{bf}$  (Figure 2b) for all the values of  $\phi$ . The ratio of the water-Au-nanofluid Rayleigh and Prandtl numbers to the base-fluid Rayleigh and Prandtl numbers decreases with the increasing volume fraction of Au nanoparticles.

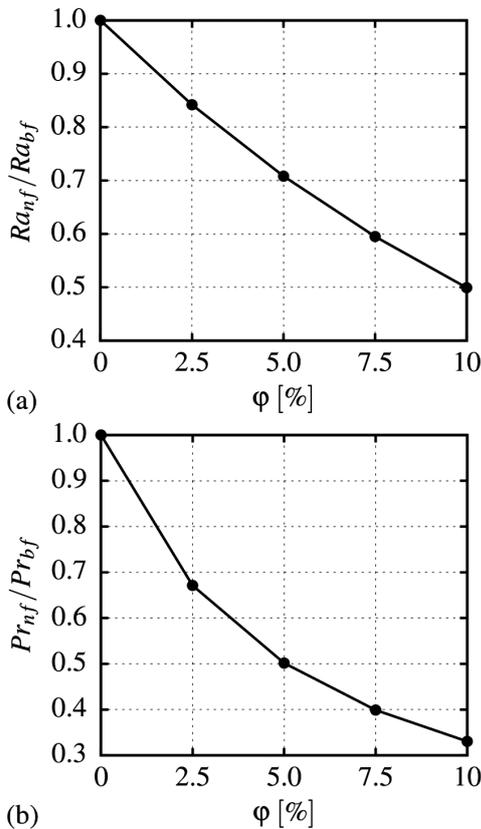


Figure 2: Variation of the dimensionless numbers of the water-Au nanofluid with the volume fraction of Au nanoparticles: a) Rayleigh number and b) Prandtl number

Slika 2: Spreminjanje brezdimenzijskih števil nanotekočine voda-Au s prostorninskim deležem Au-nanodelcev: a) Rayleighjevo število in b) Prandtlovo število

### 2.3 Grid-dependency study

The grid independence of the results has been established on the basis of a detailed analysis of three different uniform meshes:  $M1(50 \times 50)$ ,  $M2(100 \times 100)$  and  $M3(200 \times 200)$ . For the general primitive variable  $\phi$  the grid-converged (i.e., extrapolated to the zero element size) value according to Richardson extrapolation is given as<sup>11,12</sup>,  $\phi_{ext} = \phi_{M3} - (\phi_{M2} - \phi_{M3})/(r^p - 1)$  where  $\phi_{M3}$  is obtained on the basis of the finest grid and  $\phi_{M2}$  is the solution based on the next level of the coarse grid,  $r = 2$  is the ratio between the coarse- and the fine-grid spacing and  $p = 2$  is the order of accuracy.

The numerical error  $e = |(\phi_{M2} - \Phi_{ext})/\Phi_{ext}|$  for the mean Nusselt number  $\overline{Nu}$  is presented in Table 2. It can be seen that the differences in the grid refinements are exceedingly small and the agreement between mesh  $M2$  and the extrapolated value is extremely good (the discretisation error is well below 0.2 %). Based on this, the simulations in the remainder of the paper were conducted on mesh  $M2$  that provided a reasonable compromise between high accuracy and computational efficiency.

Table 2: Effect of a mesh refinement upon the mean Nusselt number ( $\phi = 0.10$ ,  $Ra_{bf} = 10^5$ )

Tabela 2: Vpliv zgoščevanja mreže na srednjo vrednost Nusseltovega števila ( $\phi = 0.10$ ,  $Ra_{bf} = 10^5$ )

Mesh $M1$	Mesh $M2$	Mesh $M3$	$\overline{Nu}_{ext}$	$e$
3.325	3.304	3.299	3.298	0.184 %

### 2.4 Benchmark comparison

In addition to the aforementioned grid-dependency study, the simulation results have also been compared with the recent results of Turan et al.<sup>14</sup> for the Rayleigh-Bénard natural convection in a square cavity. The comparisons between the present-simulation results and the corresponding benchmark values (summarised in Table 3) are very good and entirely consistent with our grid-dependency studies.

Table 3: Comparison of the present results for  $\overline{Nu}$  with the benchmark results

Tabela 3: Primerjava pričujočih rezultatov za  $\overline{Nu}$  z referenčnimi rezultati

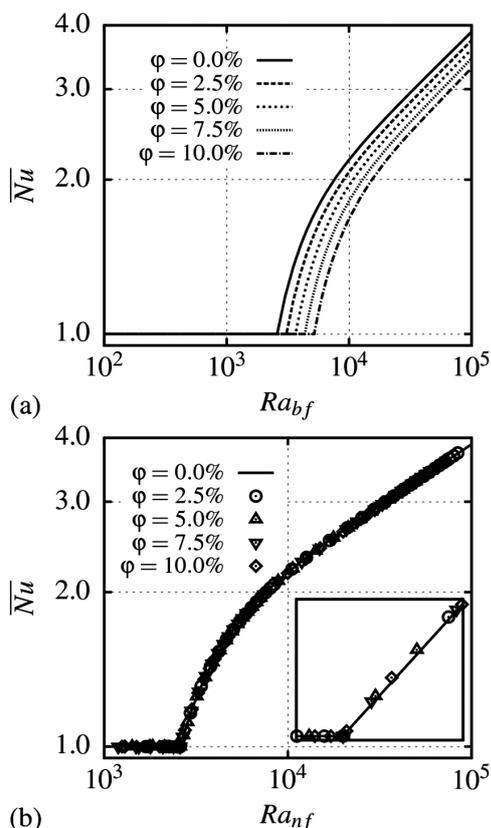
	$Ra = 10^3$		$Ra = 10^4$		$Ra = 10^5$	
	$Pr = 1$	$Pr = 10$	$Pr = 1$	$Pr = 10$	$Pr = 1$	$Pr = 10$
Present study	1.000	1.000	2.164	2.190	3.941	3.875
Turan et al. <sup>14</sup>	1.000	1.000	2.162	2.188	3.934	3.868

## 3 RESULTS AND DISCUSSION

Figure 3 presents the variation of the mean Nusselt number (equation 4) along the hot wall for different values of  $Ra_{bf}$  and  $Ra_{nf}$ . For  $Ra_{bf} < 2586$ , there is no convection in the nanofluid or the base fluid, and the heat transfer occurs due to pure conduction, so the mean Nusselt number equals 1 and is independent of the base-

fluid Rayleigh number (**Figure 3a**). As the base-fluid Rayleigh number increases, the nanofluid remains in the conductive regime, while convection appears in the base fluid. The point of transition (i.e., the value of  $Ra_{bf}$ ) from conduction to convection depends on the volume fraction of Au nanoparticles. The higher is the value of  $\varphi$ , the more delayed is the onset of convection (**Figure 3a**). When the nanofluid is in the convective heat-transfer regime, the mean Nusselt number is a monotonic increasing function of  $Ra_{bf}$ .

On the other hand, it is interesting to notice that the transition from conduction to convection occurs at the same value of the nanofluid Rayleigh number, i.e.,  $Ra_{nf} \approx 2586$  (**Figure 3b**). Furthermore, the value of the mean Nusselt number at a given  $Ra_{nf}$  is practically independent of the nanoparticles' volume fraction. This finding is a reflection of the nanofluid Prandtl number values considered in the present study. Its value varies (decreases with the increasing) from  $Pr_{nf}(\varphi = 0\%) = 6.84$  to  $Pr_{nf}(\varphi = 10\%) = 2.26$  and for this range of the Prandtl-number values ( $Pr > 1$ ) the relative balance between viscous and buoyancy forces is modified, so that the heat transport in the thermal boundary layer gets only marginally affected<sup>14</sup>. This marginal modification is reflected in a weak



**Figure 3:** Variation of the mean Nusselt number along the hot wall with the: a) base-fluid Rayleigh number and b) nanofluid Rayleigh number

**Slika 3:** Spreminjanje srednjega Nusseltovega števila vzdolž tople stene z: a) Rayleighjevim številom nosilne tekočine in b) Rayleighjevim številom nanotekočine

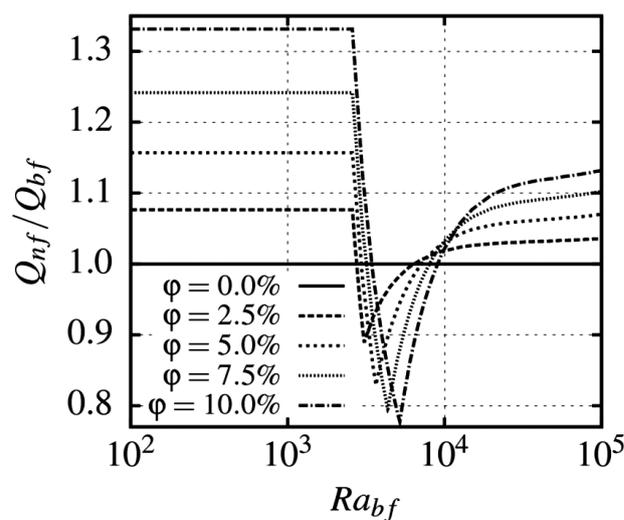
Prandtl-number (and therefore nanoparticles' volume fraction) dependence of the mean Nusselt number.

**Figure 4** shows the effect of the base-fluid Rayleigh number on the ratio of the heat-transfer rate for the water-based Au nanofluid for different values of the volume fraction. In the range  $Ra_{bf} < 2586$  the heat transfer occurs by pure conduction, so the ratio of heat transfer is equal to the ratio of thermal conductivities and is constant and independent of  $Ra_{bf}$ . For  $Ra_{nf} < 2586$  and  $Ra_{bf} > 2586$  the nanofluid remains in the conductive regime, while convection appears in the base fluid. The heat transfer is more important in the base fluid than in the nanofluid and the ratio  $Q_{nf}/Q_{bf}$  is on a decrease until  $Ra_{nf} \geq 2586$ . From this point onwards (i.e., the transition from conduction to convection) the ratio  $Q_{nf}/Q_{bf}$  is on an increase and its value becomes higher than 1, but remains lower than the ratio that is obtained when both the nanofluid and the base fluid are in the conductive regime. When the ratio  $Q_{nf}/Q_{bf} > 1$ , the heat-transfer rate in the nanofluid becomes higher than that in the base fluid.

Finally, in **Figure 4**, we observe that the heat transfer can decrease or increase depending on the value of the base-fluid Rayleigh number. For example, for a water-based Au nanofluid and for  $\varphi = 10\%$ , the ratio of the heat-transfer rate becomes higher than 1 when the base-fluid Rayleigh number reaches the value of around 9500, so we obtain an enhancement of the heat transfer only after this value ( $Ra_{bf} > 9500$ ). Therefore, adding nanoparticles increases the heat transfer only for a given value of the temperature difference.

#### 4 CONCLUSIONS

In the present study, the steady laminar natural convection of water-based Au nanofluids in a square enclosure



**Figure 4:** Effect of the base-fluid Rayleigh number on the ratio of heat transfer

**Slika 4:** Vpliv Rayleighjevega števila nosilne tekočine na razmerje prenosa toplote

sure with differentially heated horizontal walls and with the bottom wall at a higher temperature has been numerically analysed. The effects of the base-fluid Rayleigh number ( $10^2 \leq Ra_{bf} \leq 10^5$ ) and the solid volume fraction ( $0\% \leq \varphi \leq 10\%$ ) on heat-transfer characteristics have been systematically investigated in detail.

The influence of a computational grid refinement on the present numerical predictions was studied throughout the examination of the grid convergence at  $Ra_{bf} = 10^5$  and  $\varphi = 10\%$ . By utilizing extremely fine meshes the resulting discretisation error for  $\overline{Nu}$  is well below 0.2%.

The numerical method was validated for the case of Rayleigh-Bénard natural convection in a square cavity, for which the results are available in the open literature. A remarkable agreement of the present results with the benchmark results of Turan et al.<sup>14</sup> yields sufficient confidence in the present numerical procedure and its results.

Highly accurate numerical results pointed out some important points such as:

- In the classical Rayleigh-Bénard configuration, just after the onset of convection, there is more heat transfer in the base fluid than in the nanofluid. For a fixed value of the base-fluid Rayleigh number  $Ra_{bf}$ , the nanofluid Rayleigh number  $Ra_{nf}$  decreases with the volume fraction of nanoparticles. Thus, the nanoparticles delay the onset of convection.
- In the convective heat-transfer regime the mean Nusselt number  $\overline{Nu}$  is found to increase with the increasing values of the base-fluid Rayleigh number  $Ra_{bf}$ , but the  $\overline{Nu}$  values obtained for the higher values of the nanoparticles' volume fraction  $\varphi$  are smaller than those obtained in the case of the base fluid ( $\varphi = 10\%$ ) with the same numerical values of  $Ra_{bf}$ .
- The transition from the conductive to convective heat-transfer regime occurs at the same value of the nanofluid Rayleigh number, i.e.,  $Ra_{nf} \geq 2586$ .
- The values of the mean Nusselt number at a given  $Ra_{nf}$  are practically independent of the nanoparticles' volume fraction.
- The heat transfer can decrease or increase depending on the value of the Rayleigh number. So, an addition of nanoparticles increases the heat transfer only for the given values of the temperature difference.

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